PRACTICAL TREATISE
ON
MILL-GEARING.

THOMAS BOX.

MILL-GEARING.

A

PRACTICAL TREATISE

ON

MILL-GEARING,

WHEELS, SHAFTS, RIGGERS, ETC.,

FOR

THE USE OF ENGINEERS.

BY THOMAS BOX,

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PREFACE.

THE circumstances which have led to the preparation of the following work may be briefly explained. The author was extensively engaged for many years in the practical construction of Mill-gearing; he found the existing rules for calculating the Power of Wheels and Shafts very unsatisfactory; and as to Riggers, the only known rules are of comparatively modern date, and are given in such a form as to be almost useless for practical purposes. Under these circumstances, he was led to search for Rules and to construct Tables for his own daily use, and he now gives the results to the world. It is, perhaps, necessary to say thus much, because the rules are for the most part novel, and it may be well to explain that they have not been adopted in pursuance of any favourite theory, but are the result of the inflexible teachings of a long and varied experience. For the same reason, many examples are given of Wheels, Shafts, Riggers, &c., in practice, that the reader may see how far the rules agree therewith; in fact about one-third of the Tables in the work are devoted to that special purpose. As many of the rules were empirical, and did not admit of a theoretical demonstration, the only course open for proving their correctness, was by giving examples of successful application.

SANDOWN, *July*, 1877.

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PRACTICAL TREATISE ON MILL-GEARING.

CHAPTER I.

ON MOTIVE POWER.

(1.) "*Standard Unit of Power.*"—The standard unit of power in this country is the foot-pound, or the power required to raise 1 lb. avoirdupois 1 foot high; but among practical engineers the unit most commonly used is the "horse-power," being the power necessary to raise 33,000 lbs. 1 foot high per minute, or, in other words, 33,000 foot-pounds per minute.

This is James Watt's old standard, and its application appears to be very simple, but in practice certain allowances must of necessity be made for the loss by friction, &c., of the engine itself, and by the pumps or other machinery by which the work has to be done, and this fact has led to the use of such terms as "nominal horse-power," "indicated horse-power," "gross horse-power," "net horse-power," &c., which complicate the matter extremely, and cause frequent misunderstandings even among engineers themselves. We shall have to use principally the term "nominal horse-power" in this work, and it will be well at the outset to explain what we understand by that term, and its relation to the others we have named.

(2.) "*Nominal and Indicated Horse-power.*"—The reputed or "nominal" horse-power of a steam-engine is usually estimated by the net useful work to be done by it, and not by the actual power expended in doing that work, which must always include the friction of the steam-engine itself, and also that of the particular machinery employed. Thus, say that we have an engine raising 200 gallons per minute 330 feet high, by a set of deep-well pumps. The weight of a gallon of water being 10 lbs., this

is equal to $2000 \times 330 \div 33{,}000 = 20$ horse-power, and to do this work we should employ an engine of 20 *nominal* horse-power. But pumps of this kind yield in useful effect only about two-thirds or $\cdot 66$ of the power absorbed by them as shown by Table 1, we should therefore require $20 \times 3 \div 2 = 30$ horse-power to do the work assigned, and if an "indicator" were placed on the cylinder, the diagram would have to show 30 *net indicated horse-power*, clear of the friction of the engine itself. To complete the illustration, say that the friction of the engine alone was equal to 10 horse-power by the indicator, and we should then have 20 *nominal*, 30 *net indicated*, and 40 *gross indicated* horse-power.

(3.) In this case the net indicated horse-power is 50 per cent. in excess of the nominal, and the gross indicated is double the nominal; and this may be taken as a fair average allowance according to the practice of our leading engineers. Unfortunately the practice of different engineers is so very variable, that the term "nominal horse-power" has become quite an indefinite one; thus Fairbairn tells us that some of his engines nominally of 200 horse-power show 600 gross indicated horse-power, equal to 300 nominal by our ratio. Again, the engines of the 'Hercules,' by Penn, nominally of 1200 horse-power, show 8000 gross indicated, equal to 4000 nominal by our standard.

We shall use the term "nominal horse-power" in deference to custom; to substitute the more precise and definite term, "indicated horse-power," would render the book useless or nearly so to most practical men, by whom the old term is almost universally employed. By reducing the indicated to the nominal power by a certain fixed ratio in the way we have illustrated, we may obtain the precision of the former term with the convenience of the latter.

(4.) It will therefore be assumed in this work that the *net* indicated horse-power of a steam-engine is 50 per cent. in excess of the nominal, and the gross indicated, double the nominal. The indicated horse-power is 33,000 foot-pounds, but an engine of 1 nominal horse-power should yield $33{,}000 \times 1 \cdot 5 = 49{,}500$ foot-pounds clear of its own friction, and should show by the indicator $33{,}000 \times 2 = 66{,}000$ foot-pounds gross. The ratios

TABLE 1.—Of the MODULUS of MACHINES, being the duty or work done by power = 1·0.

Machines for Raising Water.	Modulus.
Inclined chain-pump	·38 to ·40
Upright „	·53 „ ·65
Archimedean screw	·62 „ ·70
Deep well and mine pumps	·66
Persian wheel, with swinging buckets	·60
Chinese wheel, with oblique-fixed buckets	·58
Endless chain and buckets, lift 3 ft. 3 in. to 6 ft. 6 in. ..	·48
„ „ 8 „ 2 „ „ 8 „ 6 „ ..	·57
„ „ 9 „ 10 „ „ 10 „ 10 „ ..	·63
„ „ 11 „ 9 „ „ 13 „ 0 „ ..	·66
Centrifugal pump, 5 feet lift, large size, 10,000 gallons ..	·75
„ 5 „ small 1,400 „ ..	·65
„ 30 „ large 1,400 „ ..	·55
„ 30 „ medium 400 „ ..	·45
„ 30 „ small 100 „ ..	·25
Water-wheels and Turbines.	
Good overshot with race	·70 „ ·80
Common overshot	·60 „ ·70
Breast wheel	·50 „ ·70
Common undershot	·30
Poncelet undershot, falls under 4 ft.	·65
„ „ 4 ft. to 5 ft.	·60
„ „ 6 „ „ $6\frac{1}{2}$ „	·55
Turbine, with water-gate full open	·75
„ „ $\frac{9}{10}$ „	·748
„ „ $\frac{8}{10}$ „	·74
„ „ $\frac{7}{10}$ „	·714
„ „ $\frac{6}{10}$ „	·675
„ „ $\frac{5}{10}$ „	·62
„ „ $\frac{4}{10}$ „	·55
„ „ $\frac{3}{10}$ „	·46
„ „ $\frac{2}{10}$ „	·346
„ „ $\frac{1}{10}$ „	·19

1, 1·5, 2, may be used for reducing the net or gross indicated power when they are known, to the standard nominal horse-power adopted as a basis in this work, and to which alone our rules apply: thus, 60 net indicated, is equal to 60 ÷ 1·5 = 40 nominal, and 60 gross indicated becomes 60 ÷ 2 = 30 nominal horse-power.

(5.) It will be observed that in (2) the nominal horse-power was identical with the real work done, the friction of the pumps, &c., being compensated by the excess of the indicated power over the nominal. But this would not hold true in every case, and with all kinds of machinery, in fact, it will apply without correction only to those cases where the machinery wastes in friction, &c., 33 per cent. of the power employed upon them, or where the modulus is ·66, and Table 1 shows that this is about the modulus of many kinds of pumping machinery.

With other machinery having a modulus more or less than ·66, the nominal power of the engine will not agree exactly with the useful work done; for instance, if we had to raise 1400 gallons 33 feet high per minute, the useful work done is $14{,}000 \times 33 \div 33{,}000 = 14$ horse-power, and with pumps of any kind whose modulus was ·66, this would be the nominal power of the engine also. But say we had to do this work by a centrifugal pump whose modulus by Table 1 is ·55; then to do 14-horse net work we require $14 \div \cdot 55 = 25 \cdot 5$ net indicated horse-power, which by the ratio we have given would require an engine of $25 \cdot 5 \div 1 \cdot 5 = 17$ nominal horse-power.

Again, to raise 100 gallons 50 feet high per minute by a small centrifugal pump whose modulus by Table 1 is ·25, we should require $1000 \times 50 \div (33{,}000 \times \cdot 25) = 6$ net indicated, or $6 \div 1 \cdot 5 = 4$ horse-power nominal, although the useful work done is only $1000 \times 50 \div 33{,}000 = 1\frac{1}{2}$ horse-power.

(6.) Before calculating the sizes for wheels, shafts, &c., by the rules in this work, the real or indicated power to be dealt with should be ascertained or carefully estimated, and the nominal horse-power deduced from it by the ratios we have given (4). When nothing is known but the reputed power of an engine, &c., of course nothing more can be done than to take that as the nominal horse-power in fixing the proper sizes for the gearing; but when the net indicated power is given, we have much more satisfactory data to work from, and we can easily reduce the indicated to nominal horse-power by taking the latter at two-thirds the former; thus, 120 net indicated horse-power would be $120 \times 2 \div 3 = 80$ horse nominal by our standard, and the gearing should be calculated for that power.

(7.) When the *gross* indicated power alone is known, which is usually the case with marine engines, the nominal may be found with moderate accuracy by taking it at half the gross indicated. Although this is much less precise and satisfactory than the result obtained from the *net* indicated power, still it is very much better than the reputed or nominal power given by the maker, as shown by the cases in (85).

(8.) "*Power of Steam-engines.*"—In cases where the particulars of an engine, such as diameter of cylinder, speed of piston, &c., are known, the nominal horse-power may be found by empirical formulæ with considerable precision, and should be used in preference to the reputed nominal power.

Let d = diameter of the cylinder in inches.
V = Velocity of piston in feet per minute (mean).
H = Nominal horse-power by our standard in (4).
M = Multiplier for the particular kind of engine, pressure, &c.

Then, we have the rules:

$$H = d^2 \times V \div M.$$
$$d = \sqrt{(H \times M \div V)}.$$
$$V = H \times M \div d^2.$$
$$M = d^2 \times V \div H.$$

The value of M will vary with the kind of engine; for common high-pressure engines, with 45 lbs. of steam in the boiler, acting without expansion, except a small amount obtained by lap of slide, the steam being cut off thereby at about seven-tenths of the stroke; the mean value of M may be taken at 2250.

With the same pressure, the steam being cut off at one-third by an expansion slide and acting expansively, non-condensing, M = 3400.

With the same pressure, steam cut off at one-fourth, acting expansively and condensing, M = 4000.

With the same pressure a "Woolf" engine, with two cylinders having cubic capacities in the ratio of four to one, the steam acting with its full pressure in the small cylinder, passing thence into the large cylinder where it acts expansively, and is

afterwards condensed; taking d in the rule as the diameter of the *large* cylinder, the mean value of M = 4700.

With a common low-pressure, condensing engine, the boiler pressure being about 7 lbs., M = 6400.

Table 2 gives the particulars of different kinds of engines from cases in practice, col. 3 being calculated by the rules; the value of M for each case as calculated by the fourth rule is given by col. 8.

(9.) These rules may be used to find the proper diameter of cylinder for a given nominal horse-power: say we require the diameters necessary for the different kinds of engine, all of 30 nominal horse-power, with velocity of piston = 220 feet per minute. Then the rule $d = \surd\ (H \times M \div V)$ becomes for a common high-pressure engine $(30 \times 2250 \div 220)\ \surd = 17\frac{1}{2}$ inches diameter of cylinder; with a high-pressure, expansive, non-condensing engine $(30 \times 3400 \div 220)\ \surd = 21\frac{1}{2}$ inches; with a high-pressure, expansive, condensing engine $(30 \times 4000 \div 220)\ \surd = 23\frac{3}{8}$ inches; with a Woolf or double-cylinder engine $(30 \times 4700 \div 220)\ \surd = 25\frac{3}{8}$ inches diameter of large cylinder; and with a common low-pressure, condensing engine $(30 \times 6400 \div 220)\ \surd = 29\frac{1}{2}$ inches diameter. Of course it must be understood, that in all these cases the pressure of steam, rate of expansion, &c., shall be about the same as is assumed in the rules.

(10.) When the pressures, &c., differ from those assumed in (8) some modification of the rules is required. Putting p for the *mean* pressure on the piston, including the vacuum in condensing engines, and the rest as before, we then have the rules:

$$H = d^2 \times p \times V \times M';$$
$$p = H \div (d^2 \times V \times M').$$

We now require only two values of M′, namely, for non-condensing and condensing engines respectively; for the former its mean value may be taken at ·000012, and for the latter at ·0000082.

Thus, with a common high-pressure engine, say $d = 9\frac{1}{2}$, V = 150, with say 45 lbs. steam in the boiler, reduced by friction of steam-pipe, &c., to 40 lbs. at the engine, cut off at

TABLE 2.—Of the NOMINAL HORSE-POWER of STEAM-ENGINES of different kinds and sizes:—Cases in Practice.

No.	Nominal Horse-power.	Nominal Horse-power by Rule in (8).	Diameter of Cylinder.	Stroke.	Revolution per Minute.	Velocity of Piston, ft. per Minute.	Value of M.	Nominal Horse-power by Rule in (10).
			inches.	ft. in.				
			COMMON LOW-PRESSURE, CONDENSING ENGINES.					
1	80	79·6	45½	7 0	17·6	246	6365	79·4
2	45	44·2	34¾	6 0	19·5	234	6282	44·0
3	25	27·8	28	5 6	20·6	227	7120	27·7
4	12	12·1	19½	4 0	25·5	204	6460	12·1
5	4	4·3	12½	2 6	35·0	175	6838	4·3
			WOOLF, OR DOUBLE-CYLINDER ENGINES.					
6	100	100·8	46	8 0	14·0	224	4700	..
7	60	61·0	35	6 0	19·5	234	4777	..
8	42	42·0	30	5 0	22·0	220	4700	..
9	35	38·0	32½	5 0	22·0	220	5104	..
10	30	26·4	23¾	4 6	24·5	220	4136	..
11	22	22·0	22	4 3	25·5	217	4774	..
12	12	12·8	17¾	2 10	34·0	193	5067	..
			HIGH-PRESSURE, EXPANSIVE, CONDENSING.					
13	45	45·0	30	2 6	40·0	200	4000	45·7
14	20	20·0	20	2 0	50·0	200	4000	20·3
15	15	16·2	18	2 0	50·0	200	4320	16·5
16	12	12·0	15½	2 0	50·0	200	4000	12·2
17	10	9·0	15	2 0	40·0	160	3600	9·1
18	9	8·5	13	2 0	50·0	200	3756	8·6
			HIGH-PRESSURE, EXPANSIVE, NON-CONDENSING.					
19	25	25·5	19	3 0	40·0	240	3445	25·0
20	10	10·1	12	2 0	60·0	240	3445	10·0
21	4	4·0	8	1 2	90·0	210	3360	3·9
			COMMON HIGH-PRESSURE, NON-EXPANSIVE.					
22	10	9·4	11½	2 0	40·0	160	2112	9·4
23	8	7·8	10½	2 0	40·0	160	2200	7·8
24	6	6·0	9½	1 8	45·0	150	2250	6·0
25	4	4·0	8	1 4	53·0	142	2272	4·0
26	1	1·1	4½	0 10	72·0	121	2450	1·1
27	½	0·5	3½	0 7	91·0	106	2239	0·5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

seven-tenths by lap of slide-valve, which by Table 3 is equal to 37 lbs. mean pressure throughout the stroke:—then by the rule we have $9\frac{1}{2}^2 \times 37 \times 150 \times \cdot 000012 = 6$ nominal horse-power; see No. 24 in Table 2, col. 9.

Again, with a high-pressure, expansive, non-condensing engine, say $d = 19, V = 240$, with 45 lbs. of steam in boiler, reduced to 40 lbs. at the engine, cut off by expansion slide at one-third of the stroke, by which the mean pressure is reduced by Table 3 to 23·5, say 24 lbs.; then by the rule we obtain $19^2 \times 24 \times 240 \times \cdot 000012 = 24 \cdot 95$, say 25 horse-power; see No. 19 in Table 2, col. 9.

Again, with a high-pressure, expansive and condensing engine, say $d = 20, V = 200$; 45 lbs. steam in the boiler reduced to 40 lbs. at the engine, cut off at one-fourth the stroke by expansion slide, and thus reduced by Table 3 to 17·8, say 18 lbs. mean pressure, which added to say 13 lbs. vacuum, gives $p = 18 + 13 = 31$ lbs., and we have $H = 20^2 \times 31 \times 200 \times \cdot 0000082 = 20 \cdot 3$ nominal horse-power; see No. 14 in Table 2, col. 9.

Again, with a common low-pressure, condensing engine, say $d = 45\frac{1}{2}, V = 246$, with 7 lbs. steam in the boiler, reduced to 6 lbs. mean pressure above atmosphere in the cylinder, which added to 13 lbs. vacuum, gives $p = 6 + 13 = 19$ lbs., and H becomes $45\frac{1}{2}^2 \times 19 \times 246 \times \cdot 0000082 = 79 \cdot 4$ nominal horse-power; see No. 1 in Table 2, col. 9.

(11.) The rule $p = H \div (d^2 \times V \times M')$ is useful for determining with each class of engine the pressure necessary to obtain a given power. Thus we found with a high-pressure, expansive, condensing engine when $d = 20, V = 200$, that 45 lbs. in the boiler gave 20·3 horse-power; but say that we required to work this engine up to 30 nominal horse-power, and desire to know the increased pressure necessary to effect that purpose. Then, the rule $p = H \div (d^2 \times V \times M')$ becomes $30 \div (20^2 \times 200 \times \cdot 0000082) = 46$ lbs. mean pressure throughout the stroke, and the vacuum being constant and equal to 13 lbs., the steam must yield $46 - 13 = 33$ lbs. per square inch, and being cut off at one-fourth the stroke, its initial pressure by Table 3 must be 65 lbs. at the engine, or say 70 lbs. in the boiler, &c.

TABLE 3.—Of the MEAN PRESSURE of EXPANSIVE STEAM.

Initial Pressure above Atmosphere in lbs. per Square Inch.	Point of Stroke at which the Steam is cut off.											
	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{1}{3}$	$\frac{4}{10}$	$\frac{1}{2}$	$\frac{6}{10}$	$\frac{2}{3}$	$\frac{7}{10}$	$\frac{3}{4}$	$\frac{8}{10}$
	Mean Pressure of Steam throughout the Stroke.											
15	−5·1	·66	2·9	4·8	6·0	8·0	10·4	12·0	13·1	13·4	14·0	14·3
20	−3·5	3·27	5·9	8·1	9·5	11·8	14·6	16·7	17·6	18·2	18·8	19·2
25	−1·8	5·88	8·8	11·4	13·0	15·6	18·8	21·2	22·3	22·9	23·6	24·1
30	−0·15	8·5	11·8	14·7	16·5	19·5	23·0	25·8	27·0	27·7	28·4	29·0
35	1·50	11·1	14·8	18·0	20·0	23·3	27·3	30·3	31·6	32·4	33·2	33·9
40	3·15	13·7	17·8	21·3	23·5	27·1	31·5	34·8	36·3	37·2	37·6	38·8
45	4·80	16·3	20·8	24·6	27·0	31·0	35·8	39·4	41·0	41·9	42·9	43·7
50	6·45	19·0	23·8	27·9	30·5	34·8	40·0	43·9	45·6	46·7	47·2	48·6
55	8·10	21·5	26·7	31·2	34·0	38·6	44·2	48·4	50·3	51·4	52·5	53·5
60	9·75	24·1	29·7	34·5	37·5	42·5	48·5	53·0	55·0	56·2	57·4	58·4
65	11·4	26·7	32·7	37·8	41·0	46·3	52·7	57·5	59·6	60·9	62·2	63·2
70	13·0	29·4	35·7	41·1	44·5	50·1	57·0	62·0	64·3	65·7	66·0	68·1
75	14·7	32·0	38·7	44·4	48·0	54·0	61·2	66·5	69·0	70·4	71·9	73·1
80	16·3	34·6	41·7	47·7	51·5	57·8	65·5	71·1	73·6	75·2	76·6	78·0
90	19·6	39·8	47·7	54·3	58·5	95·5	73·9	80·1	83·0	84·6	86·3	87·7
100	23·0	45·0	53·7	60·9	65·5	73·1	82·4	89·2	92·3	94·1	96·0	97·5
120	29·5	55·4	65·5	74·1	79·4	88·5	99·3	107·0	111·5	113·2	115·6	117·0
150	39·5	71·1	83·4	94·0	100·4	111·5	124·7	134·2	139·6	141·6	144·7	146·4

(12.) "*Power of Water.*"—Taking the case of a water-wheel driving a set of pumps, we may take the same pumps as before (2), where the net useful work done was 20 horse-power; we should estimate the power of the wheel at 20-horse nominal, and the gearing might be calculated for that power by the rules in this book. But the actual power expended in doing this work by pumps whose modulus is ·66 would be $20 \div \cdot 66 = 30$ horse-power or $33{,}000 \times 30 = 990{,}000$ foot-pounds, and with a good overshot water-wheel yielding 75 per cent. of the power of the water, the *gross* power of the latter would be $30 \div \cdot 75 =$

40 horse-power; here then we have 40 horse-power gross, 30-horse net, and 20-horse nominal.

(13.) The ratio of the useful work done to the gross power of the water will vary with the kind of water-wheel or turbine employed, as is shown by Table 1; thus with a common undershot water-wheel yielding only 30 per cent. of the gross power of the water, for 20-horse nominal, or 30-horse net, we should require $30 \div \cdot 3 = 100$-horse gross power of the water.

A good "turbine" should yield 75 per cent. usefully, but it will do this only when working with its full power, the water-gate being full open so as to fill the wheel completely. When open to a less extent the duty falls off considerably, and this is a drawback to the use of a turbine in cases where the work is necessarily variable. From a comparison of the duty done by turbines at Mülbach, Moussay, and Lowell we obtain the results given by Table 1.

Say that we have a mill-stream delivering 4800 cubic feet of water per minute, with a fall of 10 feet. A cubic foot of water weighing 62·3 lbs., we have $4800 \times 62 \cdot 3 \times 10 \div 33{,}000 = 90$ gross horse-power. A common breast wheel yielding 50 per cent. usefully, this is equal to 45 horse-power, as measured by a dynamometer, &c., on the water-wheel shaft, which is equivalent to 45 indicated horse-power with a steam-engine, or to $45 \div 1 \cdot 5 = 30$ nominal horse-power by our standard (4), and the gearing may be calculated for the latter power by the rules in this work.

(14.) "*Power of Men, Horses, &c.*"—The power of animals varies greatly with the mode in which it is exerted, and the duration of the labour. In the case of a horse, the most favourable mode of exerting the strength is in drawing a carriage at a low velocity, but even then is not equal to the standard of 33,000 foot-pounds per minute, except where the duration of the labour is reduced to about five hours per day. When a horse walks in a circle for the purpose of driving machinery, the effective power is reduced by the unfavourable rotary motion; the diameter of the circular path should be as large as con-

venient, not less than 20 or 25 feet if possible, and the velocity should not exceed $1\frac{3}{4}$ to 2 miles per hour. When the power is employed in raising water by pumps the useful effect is still further reduced by friction. Table 4 gives the power of men, horses, &c., all reduced to net horse-power of 33,000 foot-pounds per minute; the following examples will illustrate its application to practice.

Say we had to raise 20 gallons per minute 120 feet high for eight hours per day, by a set of deep-well pumps; the useful work done is $200 \times 120 \div 33{,}000 = \cdot 73$ net horse-power, and will require by Table 4, $\cdot 73 \div \cdot 354 = 2$ horses, or $\cdot 73 \div \cdot 053 = 14$ men working the pumps by winches.

The Table 4 may also be applied to any machinery whose modulus is known; thus, say that we have an upright chain-pump raising 200 gallons per minute 20 feet high for six hours per day; the net useful work done is $2000 \times 20 \div 33{,}000 = 1 \cdot 21$ horse-power, and taking the modulus of the pump from Table 1, at $\cdot 53$, this is equal to $1 \cdot 21 \div \cdot 53 = 2 \cdot 3$ horse-power net, and we shall require $2 \cdot 3 \div \cdot 614 = 4$ horses, or $2 \cdot 3 \div \cdot 092 = 25$ men working the pump with winches, or $2 \cdot 3 \div \cdot 109 = 21$ men with capstans, &c. It will be observed that we have here taken the direct net power of the man or the horse from Table 4, divested of the friction of the pumps, which is allowed for in the modulus $\cdot 53$.

In calculating the proper strength for the horse-works, allowance should be made for the maximum strain which a horse may exert for a few moments under the whip, when five or six times the mean power may be exerted (110).

(15.) "*Day's-work of Horses, &c.*"—It will be found by Table 4 that the total day's work of men and horses is not constant, but that it increases with the duration of the labour; for instance, with a horse walking in circle, the total day's work for two hours' duration may be represented by $1 \cdot 06 \times 2 = 2 \cdot 12$, but for eight hours per day it becomes $\cdot 531 \times 8 = 4 \cdot 248$, or double. This is a general law with all animal labour; the force exerted is or should be *inversely* proportional to the square root of the duration of the labour, the result being that the total

TABLE 4.—Of the POWER of HORSES, MEN, &c., &c.

	Velocity in Feet per Minute.	Duration of Labour in Hours per Day. 2	3	4	5	6	8	10
		Net Horse-power, Work done.						
Horse, walking in a circle	176	1·06	·867	·751	·672	·614	·531	·475
Pony or Mule ,, ...	180	·708	·578	·501	·448	·409	·354	·315
Bullock ,, ...	120	..	..	·668	·598	·545	·472	·422
Ass ,, ...	157	..	..	·208	·186	·170	·147	·131
Horse, in circle, raising water by deep-well pump ..	176	·708	·578	·501	·448	·409	·354	·315
Pony or Mule ,, ,, ..	180	·473	·386	·334	·300	·272	·236	·211
Bullock ,, ,, ..	120	..	..	·445	·398	·364	·315	·281
Ass ,, ,, ..	157	..	..	·138	·124	·113	·098	·088
Man working at a winch	220	·160	·131	·113	·101	·092	·080	·072
,, tread-wheel	30	·236	·193	·167	·149	·136	·118	·105
,, capstan	118	·189	·154	·134	·120	·109	·094	·084
,, with winch and deep-well pump	220	·107	·087	·075	·067	·061	·053	·048
,, tread-wheel ,, ...	30	·157	·128	·112	·098	·091	·079	·070
,, capstan ,, ...	118	·125	·102	·088	·079	·072	·062	·055

day's work done is *directly* proportional to the square root of the duration. Thus with

1	2	3	4	5	6	8	10

hours per day, the force exerted with a constant velocity would be in the ratio

1	·705	·577	·500	·448	·408	·353	·316

Now, multiplying the duration by the force we obtain the ratio of the day's work, which becomes

1	1·41	1·73	2·00	2·24	2·45	2·83	3·16

Table 4 has been calculated on this principle, which requires, however, some further explanation:—Thus the labour of a horse walking in a circle for five hours per day is given at ·672 horse-power, which must not be understood to mean that the horse would work without cessation for five hours and then desist for the day, for when yielding ·672 horse-power the labour is excessive, and can only be done in spells of say half an hour (more or less, as may be dictated by experience), with equal intervals of rest between each, so that the horse works five hours and rests five, and the day's labour is spread over the usual day of ten hours in every case, or twelve hours including the usual meal-times. The heavier the labour, the shorter the spell, and the longer in proportion the interval of rest; thus with two hours' labour per day he would probably work two minutes and rest eight minutes, &c.; see (65) and Table 13.

CHAPTER II.

ON WHEELS—FORM OF TEETH.

(16.) "*Pitch.*"—The pitch of a wheel is the distance from centre to centre of two contiguous teeth; but there are two ways in which this may be measured, namely, along the curved pitch-line,—and in a direct straight line from centre to centre, or in other words, along the *arc*,—and along the *chord*, and there

is a great difference of opinion as to which of the two is correct. If this question were a mere distinction or definition of terms, it would be an unimportant matter, but in the case of a small pinion working with a large wheel or with a rack, a considerable positive error ensues from the adoption of an erroneous principle, and the wheels will not work together correctly. Professor Wallace, Donkin, Templeton, and others have adopted the chord or straight line mode of measurement, and have given tables for calculating the diameter of wheels on that principle. It is a result of the chord principle, that the diameters of wheels are not exactly proportional to the number of teeth: for instance, a wheel with ten teeth is not precisely half the diameter of one with twenty teeth, &c., &c. Professor Willis, Grier, and others measure the pitch by the arc or along the curved pitch-line, and this is unquestionably the correct mode. Perhaps an *ocular* demonstration will be the most convincing, especially to practical men: let Fig. 1 represent to a scale half the full size, a pinion of ten teeth, working into a rack, the pitch being four inches, and to illustrate the matter more clearly let there be *no clearance* between the teeth; for the same reason the teeth are drawn of extra length. The form of the teeth having been accurately determined by rolling circles as explained in (18), the correctness of the principles (whatever they may be) on which the pitch is measured is manifested by the accuracy with which the teeth fit into one another, being in contact at the points A.B.C.D.E.F. If now the direct distance G.P.M be taken accurately in the compasses and compared with the pitch of the rack, it will be found to be about $\frac{1}{15}$th of an inch less than Q.R; but if the pitch of the pinion be measured along the curved line by short steps G.H.I.J.K.L.M, and the same distances be set off from Q, on the pitch-line of the rack at H′. I′. J′. K′. L′. R, the two sets of measurements will exactly coincide, and thus the method of measuring the pitch along the curved pitch-line is proved to be correct. To make the error of the other, or chord principle more palpable, let the direct distance G.P.M be set off three times from R to S.T and U, and it will be found that by the accumulation of the error, the centre of the tooth U is about $\frac{2}{10}$ths of an inch wrong in position, and that the teeth

would cut into one another to that extent at A . W as shown by the dotted line V . W . Z, and all the rest would do the same, only to a less extent.

(17.) "*Diameter of Wheels.*"—The primary idea of a pair of wheels is the case of two plain cylinders working against each other and driving by frictional contact; these are called the *pitch-circles*, and by the diameter of a wheel, the diameter of these imaginary circles is always understood. The diameter of a wheel with a given pitch and number of teeth, may be found from any table of circumferences of circles, or multiplying the pitch by the number of teeth, and dividing the product by 3·1416 will give the diameter in inches; thus a wheel with 122 teeth 2 inches pitch, must be $122 \times 2 \div 3{\cdot}1416 = 77{\cdot}66$ inches diameter at the pitch-line. The diameter may also be calculated by the constant multipliers in col. 3 of Table 5; thus in our case we have $\cdot 6366 \times 122 = 77{\cdot}66$ inches as before.

(18.) "*Form of Teeth.*"—The form of teeth should be such as to give the same perfect uniformity of motion as with plain cylinders rolling on one another. Mathematicians have shown that there are two curves which fulfil that condition, namely, the *epicycloid* and the *involute.*

The Epicycloid is a curve drawn by a point in the circumference of a circle rolling on another circle, or on a plane, which may be regarded as a circle of infinite radius. Let A and B in Fig. 2 be the pitch-circles of a pair of wheels of unequal diameters, and C a circle say half the diameter of B: in that case a pencil or point at D in the circumference of C rolling inside the circle B into the position C′ will draw a straight and radial line D.E, and that will be the form of the tooth in the wheel B *below* pitch-line. If now this same circle C be made to roll outside the circle A from the position F to the position G, a pencil or point in its circumference at H will describe the epicycloidal curve H.J which will be the form of the tooth of the wheel A *above* pitch-line. It will be observed, that in all cases, the part of the tooth of any wheel above pitch-line, works only with the part below pitch-line in its fellow, and *vice versâ;* in our case therefore the curve H.J works with the radial line D.E and will do so correctly. The teeth of the pinion A

TABLE 5.—Of the PROPORTIONS of TEETH, &c., &c., for SPUR-WHEELS.

Pitch. in Inches.	Suitable width in Inches.	Constant for the Diameter.	IRON AND IRON TEETH. Length of Teeth. Above Pitch.	IRON AND IRON TEETH. Length of Teeth. Below Pitch.	IRON AND IRON TEETH. Length of Teeth. Total.	IRON AND IRON TEETH. Thickness of Teeth.	IRON AND IRON TEETH. Clearance.	MORTISE-WHEELS. Length of Teeth. Above Pitch.	MORTISE-WHEELS. Length of Teeth. Below Pitch.	MORTISE-WHEELS. Length of Teeth. Total.	MORTISE-WHEELS. Thickness of Teeth. Wood.	MORTISE-WHEELS. Thickness of Teeth. Iron.	MORTISE-WHEELS. Metal at end of Mortise.	MORTISE-WHEELS. Depth of Rim.
1	1¾	·3183	·344	·469	·813	·45	·100	·250	·375	·625	·60	·4	·500	1¼
1¼	2½	·3979	·430	·570	1·000	·57	·112	·312	·452	·764	·75	·5	·592	1½
1½	3¼	·4775	·516	·669	1·185	·689	·123	·375	·528	·903	·90	·6	·681	1¾
1¾	4	·5570	·602	·767	1·369	·809	·132	·438	·603	1·041	1·05	·7	·768	2
2	5¼	·6366	·688	·865	1·553	·930	·141	·500	·677	1·177	1·20	·8	·853	2¼
2¼	6	·7162	·774	·951	1·725	1·050	·150	·563	·751	1·314	1·35	·9	·938	2½
2½	7	·7958	·860	1·058	1·918	1·171	·158	·625	·823	1·448	1·50	1·0	1·020	2¾
2¾	8¼	·8753	·946	1·153	2·099	1·292	·166	·688	·895	1·583	1·65	1·1	1·102	3
3	9¼	·9549	1·032	1·248	2·280	1·414	·173	·750	·966	1·716	1·80	1·2	1·183	3¼
3¼	10½	1·0345	1·118	1·343	2·461	1·535	·180	·813	1·038	1·851	1·95	1·3	1·263	3½
3½	11¾	1·1141	1·204	1·438	2·642	1·657	·187	·875	1·109	1·984	2·10	1·4	1·343	3¾
3¾	13	1·1936	1·290	1·532	2·822	1·778	·194	·938	1·180	2·118	2·25	1·5	1·421	4
4	14½	1·2732	1·375	1·625	3·000	1·900	·200	1·000	1·250	2·250	2·40	1·6	1·500	4¼
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)

NOTE.—See also the scales in Plate 4, which give the dimensions in this Table by direct measurement.

below pitch may be described on the same principles by a circle K, half the diameter of A rolling inside that circle, into the position K′, when a pencil in its circumference at D, will draw the straight and radial line D.L, and the same circle rolling outside the circle B from the position M to the position N describes the curve O.P, which will work correctly with the radial line D.L.

(19.) The teeth of mortise-wheels are commonly formed precisely in the manner we have described; the teeth have in that case to be shaped and trimmed by hand, and the parts below pitch being flat, can be worked more easily than if they were formed with a hollow curve. The line drawn by a circle rolling inside another of double its own diameter, being always a straight and radial line, of course the trouble of drawing that line by rolling circles may be avoided by using a straight-edge, &c.

(20.) Where the case is not governed by any question of practical convenience, the size of the describing circle is arbitrary. For instance, the circle K rolling from Q to R would describe the hollow curve S.T, which would work with the curve W.X, described by the same circle rolling from U to V. In that case, the whole of the working parts of the teeth in both wheels are drawn by one and the same circle K, applied at K.M.Q.U, and that circle might have been of any other size at pleasure, the form of the teeth varying with every change in the diameter, but still working together correctly.

(21.) The practical millwright can thus obtain almost any desired form of tooth to suit his judgment, by varying the sizes of the describing circles, so long as the great general principle is adhered to—that the same describing circle is used for those parts of the teeth which work together—namely, above pitch in one wheel, and below pitch in the other. It is, however, an almost imperative practical condition, that the describing circle shall not be larger than half the diameter of the smallest wheel, otherwise the teeth of that wheel would be very small at the root, and consequently very weak.

(22.) When three or more wheels, all of different sizes, work together, the case becomes more complicated, but must be treated on the same principles. Thus the wheel I, would have

its tooth above pitch, formed by the same circle C, by which the tooth of B, below pitch was formed, as at Y.Y′, and the describing circle K, which formed the curve O.P above pitch in the wheel B, will form the tooth below pitch in the wheel I, by rolling inside the pitch-circle from Z to Z′. If a constant or standard describing circle, such as K, be used, the whole matter is simplified.

(23.) The advantages of using a *constant size* of describing circle are very great: for instance, any wheel having its teeth thus formed, will work correctly with any other similar wheel, which is not the case where a variable size of describing circle is used; and again, the trouble of making templets for every different size of rolling circle, and the probability of error by the adoption of a wrong one, is avoided. But to carry this out, the "Standard" size of describing circle adopted must be very small in order to avoid making the teeth of small pinions very weak at the root, and the effect of a very small circle on medium-sized wheels is to give a form of tooth to which there are practical objections on the important points of strength and wearing, as we shall see when we come to the detail of the process of forming the teeth.

(24.) In applying these principles to practice, say to the wheel A, Fig. 2, a portion of the pitch-circle is drawn to the full size on paper, and a templet *a*, formed of wood, say $\frac{1}{10}$th of an inch thick, and shaped to the same radius, is laid upon it. A segment *b* of the describing circle C, &c., is made in the same way, and a needle, &c., is driven through the edge of it obliquely, as at *f*, Fig. 3, so that its point appearing on the lower side just coincides with the curve at *g*, the position on plan being at H in the templet *b*. The templet *a* being kept in position by the pressure of the left hand, the templet *b* is pressed against it with the right hand with force sufficient to prevent slipping, and is rolled on *a* into the position *c*, the needle at H making a clean indented mark on the paper from H to J. If the needle is fixed at a proper angle, the mark made will not be a scratch, but a clear indentation. By trial, a radius may be found which will draw a curve sufficiently near the exact epicycloid for practical purposes.

The part of the tooth below pitch-line is drawn in the same way by forming a female templet *d*, upon which a segment *e* of the describing circle K is made to roll into the position *h*, and the needle at D draws the line D.L on the paper, &c., &c.

(25.) The details of the method of drawing the teeth of wheels may be more fully illustrated by Fig. 1, in which we have a rack 4 inches pitch, working into a pinion with 10 teeth, having a diameter, by col. 3 of Table 5, equal to $1 \cdot 2732 \times 10 = 12 \cdot 732$, or, say $12\frac{3}{4}$ inches.

For a particular purpose, explained in (16), no clearance between the teeth is allowed in the figure; this, however, will not affect the *form* of the teeth, which is the matter now in hand. If we determine beforehand that the teeth of the pinion below pitch shall be radial, that form would of course be given by a describing circle *a a*, half the diameter of the pitch-circle of the pinion, or $6\frac{3}{8}$ inches, and by the principles already laid down, this circle rolling from *b* to *c* on the straight pitch-line of the rack will draw the curve *d e* or the form of the tooth of the rack above pitch-line. By trial we may now find a radius which will draw that curve with approximate correctness, and we find it to be about $6\frac{1}{2}$ inches, having its centre at *f*, about $1\frac{3}{16}$ inch below the pitch-line: this, however, will only draw the curve correctly from *e* to *h*, the rest, from *d* to *h*, may be drawn from the centre *k* (at the junction of the radius *h.f* with the pitch-line of the rack) with the radius *h k*, which is about $1\frac{1}{8}$ inch. It will often happen, as in this instance, that two radii are necessary to obtain a good approximation to the true epicycloidal curve.

(26.) We will assume for the rack that the form of tooth below pitch-line shall be straight and perpendicular to the pitch-line, which is the form that would be given by a describing circle of infinite radius, or, in other words, by a straight line, and a point in this same straight line rolling on the pitch-circle of the pinion from *m* to *n n*, will draw the curve D *p*, which is the form of the tooth of the pinion, above pitch-line. Searching for a radius by which this curve may be drawn, we find it to be about $3\frac{3}{8}$ inches, having its centre at *r*, about $\frac{1}{10}$th of an inch above the pitch-circle of the pinion; this, how-

C 2

ever, will draw only that part of the curve from *p* to *s*, leaving about $\frac{1}{4}$th of an inch from *s* to D to be drawn with a radius of about $1\frac{1}{8}$ inch, having its centre on the pitch-circle of the pinion.

(27.) It will be found that the form of tooth with epicycloidal curves varies very greatly with the sizes of describing circle used, and as they are all equally correct in principle, it may seem to be a matter of indifference what particular size of circle is adopted, and what form of tooth ensues. But there are other and practical considerations which are not contemplated in the rules by which the theoretical form of the teeth is fixed, but which it is necessary to remember as affecting the durability of the wheels and the power they are capable of transmitting.

(28.) "*Form as affecting Wear-and-tear.*"—The point of contact between tooth and tooth is always found in the line of the describing circle; for instance, in Fig. 1, it is at E, which is in the circle *a a*; the teeth will therefore be out of contact at the point *t*, where the describing circle and the point of the rack-teeth intersect. When passing the centre at G, the teeth are in contact at that point, or at the pitch-line; while the side of the tooth moves from G to *v*, the point of contact moves from *v* to *t*, the wear-and-tear is thus distributed over the surface *t v*, or more distinctly from *w* to *x*. The relative distance over which the strain is distributed governs the durability of the wheel, and is itself governed by the size of the describing circle; thus with a larger circle *g*, the point of intersection would have been at *i*, and in that case the wear would have been distributed over the reduced distance *i j*, or about one-third of the distance *t v*. In some cases this is reduced to a *point*, and the wear-and-tear becomes excessive: this is the case in Fig. 1, with the teeth of the rack below pitch; for the describing circle being one of infinite radius, it coincides with the straight pitch-line, and the teeth are in contact at A. C. D. F. While the wear on the point of the tooth of the pinion is distributed all over the surface from D to *p*, it is concentrated on the point D of that part of the rack-tooth with which it works, and although the theoretical action is perfect, the teeth of the rack would suffer

and very quickly change their form by excessive grinding wear.

(29.) To show how this may be altered by a change in the size of describing circle: say we take the circle *a a*, by which the point of the rack-tooth was drawn, and cause it to roll from *a'* to *b'* when it draws the hollow epicycloidal curve A . *c'*, which will be the new form of the tooth of the rack below pitch. The same describing circle rolling on the pitch-line of the pinion will draw the curve *d' e'*, giving the form of the tooth of the pinion above pitch. The line of contact, instead of being along the straight pitch-line as before, will now be along the describing circle G . *f'*, contact will cease at *f'*, where the describing circle and the point of the pinion tooth intersect, and the wear is distributed over the surface from *f'* to *h'* or from A to *k'* instead of being concentrated on a mere point as in the former case.

(30.) "*Form as affecting the Power of the Wheel.*"—Other things being equal, the power which a wheel can transmit depends on the number of teeth in gear at one time, and as the arc during which contact lasts and consequently the number of teeth in contact depends on the size of the describing circle, this is another important practical point for consideration in selecting the size of the circle to be used in any particular case. Referring again to Fig. 1, with the circle *a* . *a*., the arc of contact is G . *v*., but with a larger circle *g*, it is G . *j*., and if the pinion had been one of fine pitch, the result would have been that a greater number of teeth would have been in gear at the same time. This will also be shown by Fig. 7, in which with the small describing circle *a* the teeth part contact at the point *v*, but if the pinion had been of larger diameter, a larger describing circle might have been used, say *y*, and in that case the teeth would have remained in contact to the point *x* and the arc *u* . *x* being double the arc *u* . *v* the wheels would have been stronger in about the same proportion. In Fig. 1 the effect of using the small circle *a* . *a*. instead of the circle of infinite radius, is to reduce the distance in contact from G . A to G . *h'*, &c. But while a small circle thus reduces the number of teeth in contact and thereby the power of the wheel, another effect is to form a stronger tooth, or one having a greater thickness at the

root. The general effect of reducing the size of the describing circle is, 1st, to diminish the arc or number of teeth in gear at once, and thereby diminish the power of the wheel; 2nd, to increase the thickness of the tooth at the base and thereby increase the power of the wheel; and 3rd, to increase the wearing surface and the durability of the wheel. The relative amount of each of these can only be determined by trial in each case.

(31.) "*The form of Teeth to match those of an Old Wheel.*"—It frequently happens in practice, that a new wheel has to be made to work with an old one whose teeth are of unusual or irregular form. In such a case, the teeth of the new wheel may be accurately adapted to the old ones by the application of the principles already explained, (21). To do this, a few teeth of the old wheel should be taken accurately by a sharp pencil or steel scriber on a clean board held against them, &c., and the pitch-circle may be added by the trammel. A wooden templet as at *a*, Fig. 2, is laid upon it, and another having an *assumed* radius, as a describing circle is rolled upon it, and it is observed how far a mark or point in its circumference will draw a curve coinciding with the actual form of the old tooth. In all probability the size of the describing circle first assumed will not draw the required curve correctly, but it will easily be seen whether it is too large or too small, and another size must be tried until the coincidence is sufficiently near to satisfy the judgment of the operator. Supposing that this has been done for the part of the tooth *above* pitch-line in the old wheel, the describing circle so found will draw correctly the tooth of the new wheel *below* pitch-line. Then using a concave or female templet as at *d*, Fig. 2, we may find by trial in the same way, a diameter of describing circle that will draw a curve approximating to the form of the old tooth *below* pitch-line, which may then be applied for drawing the tooth of the new wheel above pitch-line, &c.

(32.) The process of forming the teeth by rolling circles is laborious, but it is followed by our best engineers; of course by giving plenty of clearance, and paring away the points of the teeth, wheels with *any form* of teeth may be made to work *somehow*, but where good work is desired it can only be attained

by attention to correct principles and the expenditure of time, care, and labour. Professor Willis has endeavoured to reduce the labour by the use of his "Odontograph" which gives by simple curves an approximation to the epicycloidal form drawn by a small standard describing circle, and having therefore the disadvantages which we have shown to belong thereto.

(33.) A careful draftsman may avoid the trouble of making *wooden* templets altogether by using a paper one in the following manner: say we take the case of the wheel I, Fig. 2; part of the pitch-circle being drawn on the paper to the full size is divided into equal spaces at 0 . 1 . 2 . 3 . 4, &c., &c., say an inch apart, more or less, and the same spaces are set off along the edge of the templet cut out of drawing-paper to the radius of the describing circle Y. Then the templet is applied to the pitch-circle, the divisions on its edge being made carefully to coincide with the divisions on the pitch-line, and at each junction, a mark is made with a sharp pencil opposite the division 0 on the templet. Thus when the two Nos. 1 are in contact a mark is made at 1′, when the two Nos. 2 are in contact the mark is at 2′, &c., &c., and in this way we get a series of points through which the curve may be drawn by hand, or from which a radius may be found for drawing the curve approximately with the compasses.

(34.) "*Involute Teeth.*"—The involute is a curve formed by the end of a cord unwinding from the circumference of a circle, and the teeth thus formed have one very valuable property which epicycloidal and other forms of teeth do not possess, namely, that the distance of centres of a pair of wheels may be varied so that the teeth are more or less deeply in gear without affecting the regularity of the motion, or the perfect working of the teeth. With epicycloidal teeth the distance of centres must be strictly preserved, and where this can be done, which is the case in most instances, that form of teeth is the best; but with crushing rolls, &c., &c., the distance of centres must be allowed to vary, and in such cases the peculiar property of the involute teeth renders them superior to any other.

(35.) Let W and P in Fig. 11 be the imaginary pitch-circles for a pair of wheels touching at the point A; having determined the proper length of the cog below pitch, set that distance off

from A to B on the *larger* wheel of the pair, and with the radius C . B draw the circle D . B . E ., which will be the generating circle from which the involute forming the teeth in that wheel is to be drawn. Draw the line A . H . forming a tangent to the circle D . E at H, prolong it to J, and draw the circle K . L ., touching that line at M, and that circle will be the generating circle of the involute forming the teeth of the pinion P. A string unwinding from the circle D . E . will draw the involute curve *f* . *f*, and similarly unwinding from the circle K . L . will draw the involute R . S forming the tooth of the pinion. It will answer the same purpose if a straight-edge be made to revolve against the same generating circles, a point in its edge will draw the same involute curves. The trouble of making templets of the generating circles may be avoided by making equal and corresponding divisions on the straight-edge and generating circles as explained in (33). The length of the tooth *t* . *u* should not exceed the distance L . P ., and similarly the length *v* . *x* . must not exceed the distance E . W .; they may, of course, be as much shorter than that as may be thought proper.

(36.) It will be observed, that with involute teeth, the curve is a continuous one from base to point. The line of contact, and the direction of the pressure of the teeth on each other, is in the straight line H . A . J, which in small wheels, at least, is very oblique to the direction of motion: this causes a great pressure on the axes, tending to separate the wheels, and resulting in considerable extra friction on the bearings, and consequent loss of power. This is the great objection to the involute teeth, and for this reason they should seldom be used, except in cases where the distance of centres is variable, when no other form of teeth will work correctly. It should also be observed that the thickness of the teeth should be regulated, so as to be equal at their *bases*, and will in most cases be unequal to the pitch-line, as in Fig. 11.

(37.) "*Teeth of Bevel-wheels.*"—The teeth of bevel-wheels must be drawn on the same principles as those of spur-wheels, the only important difference being, that the *projected* diameters, and not the real ones, must be taken. Let A., in Fig. 12, be a bevel-wheel, and B . its pinion, say with 54 and 22 teeth

respectively, $2\frac{1}{2}$ inches pitch, 6 inches wide on the face of teeth, &c. It is evident that all bevel-wheels have a maximum and a minimum diameter, for instance in our case they are A. *a* and B. *b* respectively; the reputed diameters are always the maximum ones, and thus we find the diameters by col. 3 of Table 5, to be $\cdot 7958 \times 54 = 40$ inches in the wheel, and $\cdot 7958 \times 22 = 17 \cdot 5$ inches in the pinion. These diameters must be drawn upon the board, as in Fig. 12, and the line F. G. drawn through their intersection at D. The line D. E. is perpendicular to the line F. G., and may be drawn by taking any convenient radius, and striking off right and left from the point D, the arcs *m. n.*, and from them as centres drawing other arcs *o. p.* and *v. s.* Then through their intersections the line C. D. E. may be drawn, intersecting the centre lines of the wheels at C and E. Now the *projected* radii E. D. and C. D. are to be taken in drawing the teeth, the projected pitch-line of the teeth of the wheel being D. *e*, drawn from the centre E, and that of the pinion being D. *f*, drawn from the centre C. The whole may then be dealt with precisely as if they were spur-wheels of the new and projected diameters. All the lines of the teeth, both in length and thickness, radiate from the centre F.

ON THE GENERAL PROPORTIONS OF WHEELS, &c.

(38.) "*Length of Teeth.*"—The length of teeth, and the proportions of wheels, are matters for judgment, experience, and taste. Table 5 gives some of the proportions for the teeth of wheels, Table 6 the proportions of the arms, and Plate 4 gives many particulars in a form more direct and useful for practical men, by scales which may be engraved on the back of the common slide-rule, or they may be collected on a pocket-rule specially devoted to them: they then can be applied direct to the work in hand; their use in practice will be illustrated as we proceed; but it may be stated that *the pitch* of the wheel on the different scales, taken in the compasses or by a rule, gives the required dimension direct.

With iron-and-iron-toothed wheels, the length of tooth above pitch may be $p \times \cdot 344 = l$, and below pitch $(p \times \cdot 344) + (\sqrt{p} \times \cdot 125) = L$. This gives the clearance between the point

of the tooth of one wheel, and the base of its fellow $= \sqrt{p} \times \cdot 125$, which is equal to $\frac{1}{4}$ inch in a wheel 4 inches pitch, and $\frac{1}{8}$ inch in a wheel of 1 inch pitch, &c.: a wheel 4 inches pitch has thus a tooth 3 inches long, or $1\frac{3}{8}$ above pitch, and $1\frac{5}{8}$ below pitch. See cols. 4, 5, and 6, in Table 5, and the scale E in Plate 4.

(39.) The teeth of mortise-wheels are commonly made much shorter than those which work iron with iron; the reason being that the wooden-cog has to be made much thicker than its iron fellow, in order to obtain the requisite strength, and the iron one would be excessively weak with the length ordinarily used for iron-toothed wheels. In mortise-wheels, the length of the wooden-cog and its iron fellow, above pitch, may be $p \times \cdot 25$, the clearance at the end of the tooth may be $\sqrt{p} \times \cdot 125$ as in iron wheels, and the length below pitch $(p \times \cdot 25) + (\sqrt{p} \times \cdot 125) = L$. Thus a mortise-wheel, 4 inches pitch, will have $2\frac{1}{4}$ inches for the total length of tooth, or 1 inch above pitch, and $1\frac{1}{4}$ inch below pitch, &c., as per cols. 9, 10, and 11, in Table 5, and the scale G in Plate 4.

(40.) "*Thickness of Teeth.*"—Taking first, the case of iron-toothed wheels working together with rough surfaces, as taken from the foundry, we must allow a certain clearance between tooth and tooth for errors of workmanship, and other irregularities. The amount of clearance may be taken at $\sqrt{p} \div 10$ which gives $\frac{1}{10}$th of an inch with 1 inch pitch, and $\frac{2}{10}$ths of an inch with 4 inches pitch, &c.: hence we have $t = \left(p - \frac{\sqrt{p}}{10}\right) \times \cdot 5$, and a wheel 4 inches pitch will have a tooth $(4 - \cdot 2) \times \cdot 5 = 1 \cdot 9$ inch thick, &c., as in col. 7 of Table 5, and the scale D in Plate 4.

With mortise-wheels clearance is unnecessary, the wooden-cog and its iron fellow being usually pitched and trimmed, or shaped accurately by hand. The thickness generally adopted as the dictum of experience, is $\frac{6}{10}$ths the pitch for the wood-cog, and $\frac{4}{10}$ths the pitch for the iron fellow. See cols. 12 and 13 in Table 5, and the scale B in Plate 4.

(41.) "*Width on Face of Teeth.*"—The width of face in a wheel is arbitrary, but as a matter of taste there should be a

certain proportion between the pitch and the width. A common proportion is to make the width $2\frac{1}{2}$ times the pitch, but this makes a wheel of small pitch too wide, and one of large pitch too narrow, in the judgment of most practical men. A better proportion will be given by the rule:—

$$w = p^2 \times 1 \cdot 8 \div \sqrt{p}$$

In which p = the pitch and w = the width in inches; col. 2 of Table 5 has been calculated by this rule.

The same proportions will apply to mortise-wheels; the mortise which receives the shank of the wooden-cog may be about $\frac{1}{4}$ inch narrower than the face of the cog, thus leaving $\frac{1}{8}$ inch for shoulder at each end.

The thickness of metal at the end of the mortise t' will be given by the rule:—

$$t' = \left(p + \sqrt{p}\right) \div 4$$

Col. 14 of Table 5 has been calculated by this rule, also the scale C in Plate 4; thus a wheel 2 inches pitch, and $5\frac{1}{4}$ inches wide on the face, would have a mortise 5 inches wide, and the thickness of metal at each end being ·853, or say $\frac{7}{8}$ inch, the total width would be $6\frac{3}{4}$ inches.

The thickness or depth of rim T, Fig. 9, in mortise-wheels is usually made equal to the pitch, but a better proportion is given by the rule $p + \cdot 25 = \mathrm{T}$, as in col. 15 of Table 5. For really good work the face of the casting should be rough turned before the teeth are fitted in; the expense of turning is nearly compensated by the saving of time and labour in fitting the cogs.

With iron-toothed wheels the thickness of the rim t, Fig. 7, is the same as the thickness of the tooth; the depth of the rib R. projecting inside the rim is two-thirds the pitch as given by the scale F in Plate 4.

(42.) "*Strength of Wheel-arms.*"—The number of arms to a wheel is to a great extent arbitrary, and is a matter to be decided by judgment and taste.

The breadth of the arm at the point will be given by the empirical rule:—

$$\mathrm{B} = \sqrt[2]{\left(7 \cdot 34 \times p \times w \times \sqrt{\mathrm{N}} \div \mathrm{A}\right)}$$

In which p = the pitch and w = the width of the wheel on the face in inches, N = the number of pinions to be driven by the wheel; A = the number of arms; and B = the breadth of the arm at the point in inches. Table 6 has been calculated by this rule.

This rule is based on the assumption, that the *thickness* of the main rib of the arm, for wheels with one pinion, is proportional to the pitch simply as in the second column of Table 6, but that where there are two or more pinions, and it becomes necessary to increase the strength of the arms by increasing their breadth, the thickness shall also be increased in the same proportion. This increase of thickness is necessary in order to avoid breadths that would otherwise be enormous and unsightly; for instance, the wheel Fig. 7, has a breadth of arm $= 6\frac{5}{8}$ inches for one pinion, and its thickness by Table 6 would be $1\frac{5}{8}$ inch; now if that thickness were maintained, we should require with four pinions, and consequently for four times the power (71), an arm double the former breadth, or $6\frac{5}{8} \times 2 = 13\frac{1}{4}$ inches. By the rule we obtain the breadth $(7 \cdot 34 \times 3 \cdot 5 \times 10 \times \sqrt{4} \div 6) \sqrt{} = 9\frac{1}{4}$ inches, but the thickness in that case must be increased from $1\frac{5}{8}$ to $1\frac{5}{8} \times 9\frac{1}{4} \div 6\frac{5}{8} = 2\frac{1}{4}$ inches.

This rule also supposes that every pinion carries the full power of the wheel, see (71); if a wheel driving one pinion has a power of 25-horse, then with two, three, and four pinions the power would be 50, 75, and 100-horse respectively. Thus, with four pinions the strength of the wheel, so far as the teeth are concerned, would be 25 horse-power, as calculated by the rules, but the strength of the arms should be equal to 100-horse, namely, the power of the four pinions, or that of the steam-engine or other motor.

(43.) The ratios of the breadths and thicknesses are both proportional to the fourth root of the number of pinions or $\sqrt[4]{N}$, hence with

1	2	3	4	5	6

pinions, we have the ratios

1	1·188	1·316	1·414	1·500	1·565

These ratios may be used with Table 6 for finding the breadths *and thicknesses of* arms for wheels with more than one pinion;

TABLE 6.—Of the BREADTH of ARM at the POINT, for WHEELS with SIX ARMS, DRIVING ONE PINION.

Pitch in Inches.	Thickness of Arm in Inches.	WIDTH ON THE FACE OF THE TEETH, IN INCHES.														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		BREADTH OF ARM AT THE POINT, IN INCHES.														
1	$\frac{7}{16}$	$1\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{1}{4}$											
$1\frac{1}{4}$	$\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{1}{8}$	$2\frac{1}{2}$	$2\frac{3}{4}$										
$1\frac{1}{2}$	$\frac{11}{16}$	..	$1\frac{7}{8}$	$2\frac{3}{8}$	$2\frac{3}{4}$	3	$3\frac{3}{8}$									
$1\frac{3}{4}$	$\frac{13}{16}$	..	..	$2\frac{1}{2}$	3	$3\frac{1}{4}$	$3\frac{5}{8}$	$3\frac{7}{8}$	$4\frac{1}{8}$							
2	$\frac{15}{16}$	..	..	$2\frac{3}{4}$	$3\frac{1}{8}$	$3\frac{1}{2}$	$3\frac{7}{8}$	$4\frac{1}{8}$	$4\frac{3}{8}$	$4\frac{3}{4}$	5					
$2\frac{1}{4}$	$1\frac{1}{16}$	..	..	$2\frac{7}{8}$	$3\frac{3}{8}$	$3\frac{3}{4}$	4	$4\frac{3}{8}$	$4\frac{3}{4}$	5	$5\frac{1}{4}$	$5\frac{1}{2}$				
$2\frac{1}{2}$	$1\frac{1}{8}$	..	..	3	$3\frac{1}{2}$	$3\frac{7}{8}$	$4\frac{1}{4}$	$4\frac{5}{8}$	5	$5\frac{1}{4}$	$5\frac{1}{2}$	$5\frac{3}{4}$	6			
$2\frac{3}{4}$	$1\frac{1}{4}$	..	..	$3\frac{1}{4}$	$3\frac{3}{4}$	$4\frac{1}{8}$	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{1}{4}$	$5\frac{1}{2}$	$5\frac{3}{4}$	$6\frac{1}{8}$	$6\frac{3}{8}$			
3	$1\frac{3}{8}$	..	..	$3\frac{3}{8}$	$3\frac{7}{8}$	$4\frac{1}{4}$	$4\frac{3}{4}$	5	$5\frac{3}{8}$	$5\frac{3}{4}$	$6\frac{1}{8}$	$6\frac{3}{8}$	$6\frac{5}{8}$	$6\frac{7}{8}$	$7\frac{1}{8}$	
$3\frac{1}{4}$	$1\frac{1}{2}$	..	..	..	4	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{1}{4}$	$5\frac{5}{8}$	6	$6\frac{3}{8}$	$6\frac{5}{8}$	$6\frac{7}{8}$	$7\frac{1}{4}$	$7\frac{1}{2}$	$7\frac{3}{4}$
$3\frac{1}{2}$	$1\frac{5}{8}$	..	..	..	$4\frac{1}{8}$	$4\frac{5}{8}$	$5\frac{1}{8}$	$5\frac{1}{2}$	$5\frac{7}{8}$	$6\frac{1}{4}$	$6\frac{5}{8}$	$6\frac{7}{8}$	$7\frac{1}{8}$	$7\frac{1}{2}$	$7\frac{3}{4}$	8
$3\frac{3}{4}$	$1\frac{3}{4}$	..	..	..	..	$4\frac{3}{4}$	$5\frac{1}{4}$	$5\frac{3}{4}$	$6\frac{1}{8}$	$6\frac{1}{2}$	$6\frac{3}{4}$	$7\frac{1}{8}$	$7\frac{1}{2}$	$7\frac{3}{4}$	8	$8\frac{1}{4}$
4	$1\frac{7}{8}$	..	..	..	..	5	$5\frac{1}{2}$	$5\frac{7}{8}$	$6\frac{1}{4}$	$6\frac{5}{8}$	7	$7\frac{3}{8}$	$7\frac{5}{8}$	8	$8\frac{1}{4}$	$8\frac{1}{2}$

thus, a wheel 2 inches pitch, 5 inches wide, with six arms and one pinion, requires by that table, an arm $3\frac{1}{2}$ inches wide at the point and $\frac{15}{16}$ inch thick; then with three pinions we require a breadth of $3\cdot5 \times 1\cdot316 = 4\cdot6$ or $4\frac{5}{8}$ inches, and a thickness of $\cdot9375 \times 1\cdot316 = 1\cdot23$, or $1\frac{1}{4}$ inch, &c.

When, in order to avoid excessive wear-and-tear, intermediate sizes are adopted for the wheels, as suggested in (72), the question of the strength of the arms becomes complicated, and the best course is to calculate the proper sizes for a wheel to carry the power with one pinion, the sizes of the arm for which as given by Table 6, or by the rules in (42), will be correct for the given *power of the engine* with any number of pinions. An illustration of this method of calculating the strength of arms in special cases is given in (73).

When a wheel is required to be in halves for convenience in fixing or carriage, &c., the breadth of half the double arm C, in Fig. 7, would be the same as the common arm of a wheel of half the width of face, or in our case 5 inches wide, which by Table 6 is $4\frac{5}{8}$ inches wide. The table gives the breadth of arm for wheels with six arms only, other cases must be calculated by the rule.

The breadth of arm increases as it approaches the centre; the amount of taper may be for large wheels about $\frac{1}{2}$ inch per foot in length, or $\frac{1}{4}$ inch on each side of the arm; with small wheels it may be $\frac{3}{4}$ inch per foot or $\frac{3}{8}$ inch on each side.

The thickness of the main rib B of the arm Fig. 8 should be equal to the thickness of the iron tooth, and the thickness of the cross ribs d may be one-fourth the pitch.

(44.) "*Strength of Metal round the Eye.*"—The thickness of metal in the boss of a wheel round the eye will be given by the rule:—

$$T = (p \times 7 \div 9) + (\cdot125 \times D)$$

In which p = the pitch in inches, D = the diameter of the wheel in feet, and T = the thickness of metal round the eye in inches. The scale A in Plate 4 is obtained by this rule; thus for the wheel Fig. 7, measuring with a rule from $3\frac{1}{2}$ inches pitch on one side of the scale, to 6 feet diameter on the other side, we find the required thickness to be $3\frac{3}{8}$ inches; if this wheel had been 10 feet diameter, the thickness would have

been $3\frac{7}{8}$ inches; or with 15 feet diameter, $4\frac{1}{2}$ inches, &c., &c. The proper proportions of the key and the key-boss, for strengthening the key-way, are given in Chapter V.

(45.) As a general illustration of the application of the preceding rules, we have in Fig. 7 a wheel of sixty-four teeth, working with one pinion of twenty teeth, $3\frac{1}{2}$ inches pitch, &c. The diameters will be found by the rule in (17), &c.: if we determine that the teeth of the pinion below pitch shall be radial, then *a . a* will be the diameter of the describing circle, being half the diameter of the pitch-circle *b*. This circle rolling from *c* to *e*, will draw the form of the tooth of the wheel *f . f*. The teeth of the wheel below pitch may be drawn by the circle *g*, whose diameter is arbitrary, and the same circle rolling from *h* to *k*, will draw the curve *m . m* which is the form of pinion tooth above pitch-line. If the large wheel had been a mortise one as at Fig. 9, then the teeth below pitch being radial, we must have taken the circle *n* half the diameter of the wheel instead of *g*, and the same circle must have been used instead of *h . k* for drawing the teeth of the pinion above pitch.

The arm D illustrates the best mode of bolting together a wheel that has to be made in halves. The main bolts E, two in number, on each side of the boss, should be placed as near the shaft as possible, and to effect that purpose, the arm must have deep bosses as shown: the bolts at F having less strain on them may be of smaller diameter, and should be as near the rim as possible. The wheel has in this case been divided in casting by the plate G, but better work is secured by planing the two halves and fitting them together thoroughly.

(46.) "*Weight of Wheels.*"—The weight of wheels can be accurately determined only by the number of cubic inches of metal they contain, which multiplied by ·2556 gives the weight in pounds. But this is very laborious, and as in many cases an approximation to the true weight is sufficient for practical purposes, we may give the following rule for finding it:—

$$W = \left\{D \times p \times w) + (\sqrt{D \times p \times w}\right\} \times M''$$

In which D = diameter at the pitch-line in feet, p = pitch in inches, w = width on the face in inches, W = weight in pounds, and M″ = a *multiplier* which varies with the kind of wheel,

but its mean value may be taken at 13 for spur-mortise-wheels; 12 for iron-toothed spur-wheels; 11 for bevel-mortise-wheels; and 10 for iron-toothed bevel-wheels. Thus, an iron-toothed spur-wheel 6 feet diameter, 3 inches pitch, and 8 inches wide, would weigh about $\left\{6 \times 3 \times 8) + (\sqrt{6 \times 3 \times 8}\right\} \times 12 = 1872$ lbs., or $16\frac{3}{4}$ cwt. The actual weight of wheels and the corresponding values of M″ are given by Table 7, and the weights of wheels as calculated by the rule are given by Table 8; as these are only approximate weights it will be prudent to allow a little for contingencies where the maximum weight is desired.

TABLE 7.—Of the ACTUAL WEIGHT of WHEEL-CASTINGS.

Diameter at the Pitch-line.		Pitch.	Width.	Weight.			Value of M.″	KIND OF WHEEL.
ft.	in.	inches.	inches.	cwt.	qrs.	lbs.		
6	7	$3\frac{1}{8}$	$9\frac{1}{4}$	21	2	2	11·7	Iron-toothed spur-wheels.
4	$4\frac{1}{2}$	$2\frac{3}{4}$	7	9	3	4	11·7	" "
4	$4\frac{1}{4}$	2	5	5	2	14	12·6	" "
3	$10\frac{3}{8}$	$2\frac{3}{4}$	7	8	2	16	11·66	" "
2	$8\frac{1}{2}$	2	$4\frac{5}{8}$	3	0	10	11·6	" "
2	3	$1\frac{3}{8}$	3	1	1	10	11·6	" "
2	1	$\frac{7}{8}$	$1\frac{1}{2}$	0	1	23	11·54	" "
4	$8\frac{3}{4}$	$3\frac{1}{8}$	9	12	3	2	9·84	Wood-toothed spur-wheels.
8	0	$2\frac{3}{4}$	7	18	1	0	12·3	" "
6	4	$3\frac{5}{16}$	$12\frac{3}{4}$	27	1	17	10·8	Iron-toothed bevel-wheels.
3	$11\frac{3}{4}$	$3\frac{1}{8}$	$9\frac{1}{4}$	11	0	0	9·74	" "
1	8	$1\frac{1}{2}$	$2\frac{1}{2}$	0	2	24	9·0	" "
1	$2\frac{1}{4}$	$1\frac{3}{8}$	$2\frac{1}{2}$	0	2	3	9·6	" "
7	$0\frac{3}{8}$	$3\frac{5}{16}$	$12\frac{5}{8}$	32	3	3	11·96	Wood-toothed bevel-wheels.
4	$5\frac{3}{4}$	$3\frac{1}{8}$	9	11	3	0	9·54	" "

(47.) "*Contraction of Wheels in Casting.*"—The contraction which metals experience in cooling down from their melting-points to ordinary temperatures is very considerable, and allowance has to be made for it in fixing the size of the pattern. The contraction of straight bars, such as girders, &c., is tolerably

TABLE 8.—Of the APPROXIMATE WEIGHT of WHEEL-CASTINGS.

D × p × w.	Spur-wheels.		Bevel-wheels.		D × p × w.	Spur-wheels.		Bevel-wheels.	
	Iron.	Mortise.	Iron.	Mortise.		Iron.	Mortise.	Iron.	Mortise.
	cwts.	cwts.	cwts.	cwts.		cwts.	cwts.	cwts.	cwts.
5	·78	·84	·64	·72	150	17·5	19·0	14·6	16·1
10	1·4	1·5	1·2	1·3	160	18·7	20·3	15·6	17·1
15	2·0	2·2	1·7	1·8	170	19·8	21·4	16·5	18·1
20	2·6	2·8	2·2	2·4	180	21·0	22·7	17·5	19·2
25	3·2	3·5	2·7	3·0	190	22·0	23·8	18·4	20·2
30	3·8	4·1	3·2	3·5	200	23·1	25·0	19·3	21·2
40	5·0	5·4	4·1	4·6	210	24·3	26·3	20·3	22·3
50	6·1	6·6	5·1	5·6	220	25·4	27·5	21·2	23·3
60	7·3	7·9	6·0	6·7	230	26·5	28·7	22·1	24·3
70	8·4	9·1	7·0	7·7	240	27·7	30·0	23·0	25·3
80	9·5	10·4	7·9	8·7	250	28·7	31·1	23·9	26·3
90	10·7	11·7	9·0	9·9	260	29·8	32·3	24·8	27·3
100	11·9	12·9	9·9	10·9	270	30·9	33·5	25·7	28·3
110	13·1	14·1	10·9	12·0	280	32·1	34·7	26·7	29·4
120	14·2	15·3	11·8	13·0	290	33·2	35·9	27·6	30·4
130	15·2	16·5	12·8	14·0	300	34·2	37·1	28·5	31·4
140	16·4	17·8	13·7	15·1	310	35·4	38·4	29·5	32·5

regular, and in the case of cast iron it is very nearly $\frac{1}{8}$ inch to a foot; but with wheels it is not so much, and is found by experience to be irregular and anomalous. This is shown by Table 9 from the author's 'Treatise on Heat.'

The irregularities in the *apparent* contraction arise in great part from the practice of "rapping" the pattern in the sand to make it an easy fit and enable it to be drawn out with facility. This is most influential in the case of small, heavy wheels of great width on the face; in some cases, and in rough hands, the casting of a small and wide pinion may be quite the full size of the pattern. The allowance to be made is therefore not uniform, but must be fixed by judgment; for wheels of ordinary sizes and proportions, $\frac{1}{10}$ inch to a foot is a good practical rule.

TABLE 9.—Of the EXPERIMENTAL CONTRACTION of CAST-IRON SPUR-WHEELS in CASTING.

Extreme Diameter of Wheel-casting.		Pitch in Inches.	Width of Teeth in Inches.	Contraction.		
				Total in Inches.	Per Foot.	
					Of Casting.	Of Pattern.
ft.	in.				inches.	inches.
10	$2\frac{3}{8}$	$3\frac{1}{2}$	12	1·08	·1059	·1040
6	$2\frac{9}{16}$	$3\frac{1}{4}$	9	·54	·0893	·0886
6	$1\frac{3}{8}$	$3\frac{1}{2}$	11	·375	·0613	·0610
5	$5\frac{9}{16}$	$3\frac{1}{2}$	11	·345	·0631	·0628
2	$11\frac{7}{8}$	$3\frac{1}{2}$	12	·11	·03896	·03884
2	$4\frac{15}{16}$	$3\frac{1}{4}$	9	·115	·0397	·0396

ON THE POWER WHICH WHEELS ARE CAPABLE OF CARRYING.

In considering the question of the strength of wheels, it is necessary to divide the subject into two parts; 1st, the absolute strength of the teeth to resist a statical or dead load, as in the case of the main wheel of a crane, or the rack and pinion of a sluice-gate; and 2nd, the strength of wheels in motion as in ordinary mill-gearing, where the force to be dealt with is a dynamic strain, or a force in motion. The laws governing the strength under these two conditions, differ entirely from each other.

(48.) "*Absolute Strength for Dead Load.*"—The tooth of a wheel may be regarded as a cantilever loaded at the end with so many pounds, Fig. 48, and its strength may be calculated by the ordinary rules for the strength of beams. Let Fig. 4 be the tooth of a wheel 3 inches pitch, which is made strong at the base by using a small describing circle (30), and let the width on the face be 10 inches. Evidently, we have a cantilever 2 inches deep, 10 inches wide, and 2·28 inches long, fixed at one end and loaded at the other. A bar of cast iron 1 inch square and 1 inch long, breaks with 6000 lbs. at one end; hence in our case we have $2^2 \times 10 \times 6000 \div 2 \cdot 28 = 105{,}300$ lbs. breaking weight.

(49.) For a crane or similar case, where life or limb is jeopardized by failure, the working load should not exceed $\frac{1}{10}$th of the breaking strain, or in our case 10,530 lbs. for the wheel 10 inches wide, or 1053 lbs. per inch width on the face: this is the strength of one tooth only.

Reasoning from this for other pitches, we shall observe that the length of the tooth and its thickness at the root are proportional to the pitch, or very nearly so, and the strength being as the thickness squared divided by the length, it follows that with the same width in all cases, the strength is in the simple proportion to the pitch: with pitches 1, 2, 3 inches the strengths will be in the ratio 1, 2, 3 also. Thus in Fig. 48 we have three cantilevers, the depths, lengths, and strains being all in the same ratio 1, 2, 3.

This result does not accord with practical instinct on the matter, partly no doubt from the fact that the width on the face is usually increased in simple proportion to the pitch, or in a yet higher ratio than that, see (41): thus with wheels 1, 2, 3, 4 inches pitch, if the widths are $1\frac{3}{4}$, $5\frac{1}{4}$, $9\frac{1}{4}$, and $14\frac{1}{2}$ inches respectively, the strengths are no longer as the pitch simply or 1, 2, 3, 4, but become $1\frac{3}{4}$, $10\frac{1}{2}$, $27\frac{3}{4}$, and 58, or in the ratios 1, 6, 16, and 33 nearly.

(50.) With fair-sized pinions we may always ensure that two teeth at least shall be in gear at once, especially when those teeth are formed on correct principles, but in crane-work very small pinions are often used, and then possibly only one tooth will be engaged at once. Admitting this, the absolute strength for a dead load will be given by the rule :—

$$W = p \times w \times 350$$

In which W = the safe weight on one tooth in pounds; p = the pitch, and w = the width, both in inches; thus for the tooth we considered in (48) 3 inches pitch, 10 inches wide, we obtain $3 \times 10 \times 350 = 10{,}500$ lbs. safe load for one tooth, or 21,000 lbs. with two teeth.

(51.) "*Effect of Shrouding.*"—The effect of shrouding a wheel up to the pitch-line as in Fig. 5, is to reduce the *effective* length of the tooth as a cantilever, and thereby increase the breaking

weight. The amount of this increase will vary with the form of the tooth to which it is applied. Taking first the case of a tooth of uniform thickness from the pitch-line to the root, as in Fig. 5, we shall observe that the depth of the cantilever is the same whether it breaks at the root or at the pitch-line, or anywhere between those two points. If the line of fracture coincided of necessity with the face of the shrouding or the pitch-line, the acting length of the cantilever would be reduced from $2 \cdot 28$ to $1 \cdot 032$ inch, and the strength would be proportionately increased, or more than doubled. But the line of fracture would most likely follow the irregular line *a a*, and the length may be taken at a mean between the two or $(1 \cdot 032 + 2 \cdot 28) \div 2 = 1 \cdot 65$ inch instead of $2 \cdot 28$ inches; hence the ratio is $2 \cdot 28 \div 1 \cdot 65 = 1 \cdot 38$ to 1, or say 38 per cent. increase.

(52.) But if we apply this reasoning to a tooth of the form Fig. 4 we shall observe that the depth of the cantilever varies from 2 inches at the base to $1 \cdot 414$ inch at the pitch-line: if we take the mean thickness at $(2 + 1 \cdot 414) \div 2 = 1 \cdot 707$, and the mean length as found before being $1 \cdot 65$, we have $d^2 \div l$ in our case $1 \cdot 707^2 \div 1 \cdot 65 = 1 \cdot 766$, which is almost precisely the strength of the unshrouded tooth, which was $2^2 \div 2 \cdot 28 = 1 \cdot 755$, showing that with this form of tooth, shrouding has no effect on the strength. The fact is that the form of the tooth Fig. 4 approximates to the parabola P, with which there would be equal strength throughout, that is to say, it would break as easily at the pitch-line or elsewhere as at the base, the load being at the end as we have assumed. With a tooth exactly of the parabolic form it would obviously be useless to attempt to strengthen it by shrouding.

(53.) The special value of shrouding is to strengthen the teeth of a small pinion working with a large wheel or with a rack; in such a case, the proper forms of the teeth are such, that there is great inequality of strength, the tooth of the large wheel being much wider at the base and therefore stronger than that of the pinion; by shrouding the latter, the strengths may be equalized. This may be illustrated by Fig. 1; the lengths of teeth in both rack and pinion are $1\frac{7}{8}$ inch, but the thickness at the base is 1 inch in the rack, and $\cdot 7$ inch in the pinion, hence the relative strengths are $1^2 \div 1 \cdot 875 = \cdot 533$ in the

former, and $\cdot 7^2 \div 1\cdot 875 = \cdot 2613$ in the latter, the pinion has therefore rather less than half the strength of the wheel. Now, by shrouding up to the pitch-line, the mean acting length becomes $(\frac{7}{8} + 1\frac{7}{8}) \div 2 = 1\frac{3}{8}$, and the mean thickness at that point, $(\cdot 7 + 1) \div 2 = \cdot 85$, hence the strength comes out $\cdot 85^2 \div 1\cdot 375 = \cdot 525$, or practically the same as the rack, which we found to be $\cdot 533$: the effect of shrouding is to double the strength of the pinion in this particular case; the increase is therefore 100 per cent.

It will be seen from this, that the effect of shrouding varies very much with the particular form of the teeth: with teeth broad at the base like Fig. 4 it is *nothing;* with teeth having parallel flanks like Fig. 5 it gives an increase of 38 per cent.; and with radial teeth where the thickness at the root happens to be $\frac{7}{10}$ths that at the pitch-line, it is 100 per cent.

(54.) When a wheel is shrouded right up to the points of the teeth, they cease to act as cantilevers altogether, and must fail by direct shearing strain, probably at the pitch-line. Thus, the thicknesses of teeth for 1, 2, 3 inches pitch, being $\cdot 45$, $\cdot 93$, and $1\cdot 414$ inch respectively by col. 7 of Table 5, and the shearing strain 16,000 lbs. per square inch, we have for teeth one inch wide $\cdot 45 \times 16000 = 7200$; $\cdot 93 \times 16000 = 14880$; and $1\cdot 414 \times 16000 = 22{,}624$ lbs. breaking weight respectively, or about double the strength of the same teeth unshrouded, as shown by the rule in (50), by which for teeth 1 inch wide we obtain $1 \times 350 \times 10 = 3500$ lbs.; $2 \times 350 \times 10 = 7000$; and $3 \times 350 \times 10 = 10{,}500$ lbs. breaking weight respectively. This increase, it will be observed, applies to the strong form of tooth, Fig. 4, which we found to be unaffected by shrouding up to the pitch-line; with weaker forms it would be proportionately greater; but the increase thus obtained is useless in many cases, for obviously only one wheel of a pair can be shrouded up to the points of the teeth, the other must of necessity be unshrouded, and the strength of the pair is determined by that of the weakest.

(55.) "*Absolute Strength of Wood-cogs.*"—The transverse strength of hornbeam, the wood commonly used for mortise-wheels, is about three-tenths that of cast iron (48), or 1800 lbs. for a bar 1 *inch square, 1* inch long, fixed at one end, and loaded

at the other. For convenience in shaping the cogs, they are usually made straight and radial below pitch (19); with wheels of moderate size this is practically the same as having parallel flanks. Now, for say 3 inches pitch, Table 5 gives the thickness of wood-cog = $1 \cdot 8$ inch, and length = $1 \cdot 716$ inch, hence, for 10 inches wide, we have $1 \cdot 8^2 \times 10 \times 1800 \div 1 \cdot 716 = 34{,}000$ lbs. breaking weight: we found in (48) that the strength of a *strong* iron tooth of the same pitch and width was 105,300 lbs., hence the ratio of the strength of iron and wood cogs is in this case $105300 \div 34000 = 3 \cdot 1$ to 1. But with a parallel-flanked iron tooth, the thickness by Table 5 being $1 \cdot 414$ inch, the strength is $1 \cdot 414^2 \times 10 \times 6000 \div 1 \cdot 716 = 70{,}000$ lbs., and the ratio to wooden cogs becomes $70000 \div 34000 = 2 \cdot 06$ to 1.

We may admit, as a general ratio, that the absolute strength of mortise-wheels to sustain a statical strain, or dead load, is one-third of the strength of iron-toothed wheels of the same pitch and width.

(56.) "*Wheels with very Slow Speeds.*"—We may now apply these results to wheels in motion, but will first confine ourselves to very low velocities for reasons that will presently appear. Say then we have a wheel with iron teeth as in Fig. 4, 3 inches pitch, 10 inches wide, 3 feet diameter, and only one revolution per minute. We found in (50) the safe strain on two teeth to be 21,000 lbs.; the velocity will be $3 \cdot 14 \times 3 = 9 \cdot 42$ feet per minute, and the *nominal* horse-power by (4) being 49,500 foot-pounds, we have $21000 \times 9 \cdot 42 \div 49500 = 4$ nominal horse-power: with mortise-wheels we should have by the ratio in (55) $4 \div 3 = 1 \cdot 33$ horse-power.

(57.) From this we obtain for very slow speeds, the rules:—

For iron-and-iron-toothed wheels $H = D \times R \times p \times w \times \cdot 0445$
For mortise-wheels .. .. .. $H = D \times R \times p \times w \times \cdot 01483$

In which D = diameter in feet, p = pitch in inches, w = width on face in inches, R = revolutions per minute, and H = nominal horse-power. Thus, in the cases we have just considered, we have $3 \times 1 \times 3 \times 10 \times \cdot 0445 = 4$ horse-power for the iron-toothed wheel, and $3 \times 1 \times 3 \times 10 \times \cdot 01483 = 1 \cdot 33$ horse-power for the mortise.

(58.) "*Wheels with High Speeds.*"—We have so far assumed

that the velocities were either nothing, or very low, and for such cases no doubt the rules are correct; but if we proceed to apply them to high or even moderate velocities, we obtain results which do not at all agree with experience; for instance, the wheel in (56) whose strength we calculated to be 4-horse at one revolution, and is probably about right at that, would be 40-horse at ten revolutions, and 400-horse at 100 revolutions, which would obviously be excessively too high.

We found in (49) that for a dead load, the strength of a tooth was directly as the square of the thickness at the base, and *inversely* as the length; the result being that the strength was simply and directly as the pitch. But with a force in motion the laws of impact show that the strength of beams is *directly* as the length and depth or $d \times l$, instead of $d^2 \div l$ as for dead load; thus, when the length and depth increase in the same proportion as they do in wheel teeth, for pitches 1, 2, 3, $d \times l$ becomes $1 \times 1 = 1$; $2 \times 2 = 4$; and $3 \times 3 = 9$ or 1^2, 2^2, 3^2, showing that for wheels with high speeds, and therefore subjected to impact, the strength is as the pitch squared, instead of as the pitch simply—as for dead loads.

(59.) The laws by which the velocity governs the strength of wheels are so obscure, that it is perhaps impossible to find rules based on purely theoretical principles that will give results at all agreeing with experience, and we are compelled to deduce empirical rules from wheels working well in practice. Mechanical *instinct*, corrected by experience, has guided our practical millwrights to a sufficient knowledge of the subject to enable them, with more or less success, to fix, by mere judgment, the proper proportions of gearing for the work to be done; it is probable that for every wheel whose sizes were fixed by rule, a thousand were fixed by judgment alone. Perhaps rules may appear unnecessary under such circumstances, but the fine old race of millwrights is becoming extinct, and is being succeeded by men of higher culture, who have lost much of the instinctive faculty as the natural result of increased theoretical knowledge. Moreover, there are great differences in the judgments of even our best engineers, some making their work much stronger than others, and indeed much stronger than necessary; besides, very great differences are found in the works of the same man, which

can only be accounted for by errors or variations of judgment. A rule is therefore necessary, although it be a merely empirical one, and has for its basis the very judgment which it is intended to guide.

(60.) Tables 10, 11, and 12, give numerous examples of wheels in practice, and from these we obtain the following rule for wheels at high and ordinary speeds:—

$$H = \sqrt{D \times R} \times p^2 \times w \times \mathsf{M}$$

In which, D = Diameter of the wheel at pitch-line, in feet.
p = pitch in inches.
w = width on the face of teeth, in inches.
R = Revolutions per minute.
H = Nominal horse-power by our standard (4).
M = Multiplier deduced from cases in practice.

M, is constant for teeth of the same material; its mean value may be taken at ·043 for iron-and-iron-toothed wheels, and at ·05 for wood-toothed, or mortise-wheels. The second cols. in Tables 10, 11, 12, have been calculated by these rules, and agree moderately well with the actual powers; for instance, in Table 10 the sum of all the actual powers is 1845-horse, and of all the calculated powers 1936·3: a difference of only 4·7 per cent. The examples have a wide range of power, from 1 to 777-horse, see (69); extreme cases are severe tests of empirical rules, and as ours agree with those extremes, and also with intermediate powers of 100, 200, 300 horse-power, they may be regarded as satisfactory.

The mean value of M for the iron-toothed wheels in Table 10 is strictly ·041, and for the mortise-wheels in Table 11 is ·0475: for the latter we have adopted the higher value ·05, as it agrees better with some of the larger cases in the table, which years of successful work have proved to be about correct.

It is remarkable that at high and moderate speeds mortise-wheels should be stronger than iron-toothed ones, as shown by the rules and tables; we have shown (55) that for a dead load, or very slow speed, iron teeth have three times the strength of wooden cogs.

Table 10.—Of the Power of Spur-wheels with Iron Teeth; from Cases in Practice.

Nominal Horse-power. Actual.	Nominal Horse-power. Calculated.	Revolutions per Minute.	Diameter of Wheel. ft.	Diameter of Wheel. in.	Pitch in Inches.	Width on the Face. inches.	Remarks.
300	304·	15·8	30	1½	4½	16	Fairbairn.
200	210·	19·5	24	5	4	14	”
140	138·	21·0	22	8¼	3¼	14	”
100	118·	22·7	21	0	3¼	12	”
90	118·	23·8	20	0	3¼	12	”
80	98·	22·7	21	0	3¼	10	”
60	51·1	19·5	8	0	3¼	9	Rag-engines; broke.
60	62·4	19·5	8	0	3¼	11	Made wider; worked well.
50	47·0	31·0	15	4½	2½	8	Fairbairn.
50	55·9	40·0	15	2½	2¾	7	Sawing machinery.
46	44·8	17·5	12	0	3	8	Boulton and Watt.
45	41·1	20·0	5	0	3¼	9	Three-throw pumps.
44	42·6	25·0	4	4⅝	3¼	9	Ditto Crystal Palace.
42	44·5	22·0	5	5	3¼	9	Rag-engines; paper-mill.
42	45·4	22	4	5 13/16	3¼	10	Three-throw pumps.
40	39·2	40	3	11¾	3	8	Rag-engines; paper-mill.
40	41·0	30·8	15	6	2½	7	Fairbairn.
32	30·0	19	8	10	3	6	Boulton and Watt.
30	29·6	24·5	6	9	2¾	7	Asphalte machinery.
30	35·4	22	2	9	3¼	10	Sugar-mill.
30	27·4	5·76	3	3	3½	12	”
30	26·7	30	13	1⅝	2½	5	Sawing machinery.
25	22·1	24·5	3	10	2¾	7	Rag-engines; paper-mill.
22	24·3	13·6	5	3	3	7½	Pumps, Crystal Palace.
20	20·6	36	4	2⅛	2½	6¼	Engineering machinery.
20	18·8	15·5	6	10¾	2½	6¾	Three-throw pumps.
20	23·0	7·28	6	8½	3	8½	”
20	22·0	14·1	5	9⅛	2¾	7½	”
20	20·3	36	13	4 7/16	2¼	4¼	Sawing machinery.
18	22·0	26·6	17	11	2	6	Fairbairn.
16	19·0	60	3	6¼	2¼	6	Sawing machinery.
16	15·8	32	3	0⅝	2½	6	Three-throw pumps.
12	12·1	34	1	7⅞	2½	6	Corn-mill.
12	18·0	47·6	10	0	2	5	Fairbairn.
8	10·4	45	1	3⅛	2½	5¼	Corn-mill.
8	8·8	24	3	11⅛	2	5¼	Three-throw pumps.
7	7·2	30	3	3½	2	4¼	”
6	6·5	45	2	1⅝	1¾	4½	”
5	5·5	19	3	4⅛	2	4½	”
4	4·5	16·4	3	11⅞	1¾	4¼	”
4	4·2	53	1	7⅛	1⅝	4	”
1	1·1	88	0	7	1⅜	2	American horse-works.

When the horse-power is given, and other particulars are required, the rule, $H = \sqrt{D \times R} \times p^2 \times w \times \mathbf{M}$, becomes:—

$$p = \sqrt{\left\{H \div (\sqrt{D \times R} \times w \times \mathbf{M})\right\}}$$

$$w = \quad H \div (\sqrt{D \times R} \times p^2 \times \mathbf{M})$$

$$R = \quad \left\{H \div (\sqrt{D} \times p^2 \times w \times \mathbf{M})\right\}^2$$

$$\mathbf{M} = \quad H \div (\sqrt{D \times R} \times p^2 \times w)$$

TABLE 11.—Of the POWER of SPUR-MORTISE-WHEELS; from Cases in Practice.

Nominal Horse-power.		Revolutions per Minute.	Pitch in Inches.	Diameter of Wheel.		Width on the Face.	REMARKS.
Actual.	Calculated.			ft.	in.	inches.	
42	45·4	22	2¾	18	3¾	6	Centrifugal pump.
30	26·0	24·5	2¼	17	2¼	5	"
30	31·7	45	2¼	13	10⅛	5	Papier-maché works.
25	24·5	25·5	2¼	14	3⅞	5⅛	Centrifugal pump.
22	18·1	26	2	14	10	4¾	Rope machinery.
16	20·0	26	2¼	13	4 7/16	4¼	Engineering machinery.
15	16·3	30	2	12	3⅝	4¼	Agricultural "
15	13·8	25·5	2¼	3	3 1/16	6	Three-throw pumps.
12	14·7	11	2¼	6	9	6½	"
12	17·1	33	2	12	3⅝	4¼	Crape-weaving machinery.
11	10·6	34	2	2	10⅜	5⅜	Three-throw pumps.
10	14·5	15·2	2¼	5	11⅝	6	"
10	12·7	16	2¼	5	1⅝	5½	"
8	9·6	16	2	5	1⅛	5⅞	"
8	7·0	16	2	2	11⅝	5	"
6	6·3	45	1½	6	1½	3⅜	Envelope machinery.
4	4·6	53	1½	3	0	3¼	Three-throw pumps.
4	4·4	53	1¾	1	3⅝	3½	"
3	3·6	65	1¾	0	8⅜	3½	"
1	1·3	100	1⅛	0	8¼	2½	Washing machinery.

TABLE 12.—Of the POWER of MORTISE-BEVEL-WHEELS; from Cases in Practice.

Nominal Horse-power.		Width on the Face in Inches.	THE DRIVING-WHEEL.				Pitch in Inches.			Max. Diameter of the Driven Wheel.	KIND OF WORK.
			Revolutions per Minute.	Diameter.							
Actual.	Calculated.			Maximum.	Minimum.	Mean.	Max.	Min.	Mean.		
				ft. in.	ft. in.	ft.				ft. in.	
40	46·2	$6\frac{1}{2}$	50	9 $0\frac{9}{16}$	8 $0\frac{1}{4}$	8·53	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	2 11	Centrifugal pump.
40	36·2	$5\frac{1}{2}$	148	4 $11\frac{3}{4}$	4 $1\frac{3}{4}$	4·56	$2\frac{1}{2}$	2	$2\frac{1}{4}$	2 $5\frac{1}{2}$	„
30	21·9	5	164	2 $10\frac{3}{8}$	2 $3\frac{1}{8}$	2·55	$2\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	2 $9\frac{3}{4}$	„
24	27·6	$5\frac{1}{2}$	50	7 6	6 $7\frac{3}{4}$	7·073	$2\frac{1}{2}$	$2\frac{1}{8}$	$2\frac{5}{16}$	3 3	„
20	20·2	$4\frac{1}{2}$	294	2 $6\frac{7}{16}$	1 11	2·228	$2\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	1 $9\frac{5}{8}$	„
20	27·3	5	40	9 $11\frac{3}{8}$	9 2	9·558	$2\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{3}{8}$	2 $9\frac{7}{16}$	„
20	24·5	5	140	4 $0\frac{11}{16}$	3 4	3·70	$2\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	2 $5\frac{3}{8}$	„
18	19·9	5	50	6 $7\frac{1}{2}$	5 $10\frac{1}{4}$	6·24	$2\frac{1}{4}$	2	$2\frac{1}{8}$	2 $6\frac{1}{8}$	„
				*			*			*	

NOTE.—The reputed diameter and pitch of bevel-wheels, by which they are commonly known by practical men, are the maximum ones marked *.

(61). To illustrate the application of these rules, say we take the first example in Table 10, where we had an iron-toothed wheel 30 feet $1\frac{1}{2}$ inch diameter, $4\frac{1}{2}$ inches pitch, 16 inches wide, $15 \cdot 8$ revolutions; to find $\sqrt{D \times R}$ we have $30 \cdot 1 \times 15 \cdot 8 = 475 \cdot 5$; the square root of which is $21 \cdot 8$; p^2 or $4\frac{1}{2}^2 = 20 \cdot 25$, hence to find H, the rule $H = \sqrt{D \times R} \times w \times \mathbf{M}$ becomes $21 \cdot 8 \times 20 \cdot 25 \times 16 \times \cdot 043 = 303 \cdot 7$ nominal horse-power.

Again, say we require the proper width for a spur-mortise-wheel, 6 feet diameter, $3\frac{1}{2}$ inches pitch, to carry 75 horse-power with 25 revolutions per minute. Then, for $\sqrt{D \times R}$ we have $6 \times 25 = 150$, the square root of which $= 12 \cdot 24$, and p^2 or $3\frac{1}{2}^2$ being $12 \cdot 25$, the rule $w = H \div (\sqrt{D \times R} \times p^2 \times \mathbf{M})$ becomes $75 \div (12 \cdot 24 \times 12 \cdot 25 \times \cdot 05) = 10$ inches, the width required.

Again, say we require the proper pitch for an iron-toothed spur-wheel $24\frac{1}{2}$ feet diameter, $19 \cdot 5$ revolutions, 14 inches wide, to carry 210 horse-power. Then to find $\sqrt{D \times R}$, we have $24 \cdot 5 \times 19 \cdot 5 = 477$, the square root of which is $21 \cdot 84$, and the rule $p = \sqrt{\left\{ H \div (\sqrt{D \times R} \times w \times \mathbf{M} \right\}}$ becomes $\left\{ 210 \div (21 \cdot 84 \times 14 \times \cdot 043 \right\} \sqrt{\ } = 4$ inches, the pitch sought.

(62.) "*Combination of the Rules.*"—On comparing the results of the rules for very low speeds in (57) with those for high and ordinary speeds in (60), it will be found that with each particular pitch there is a certain speed, usually a very low one, at which they coincide, so that the strength of the wheel at that speed may be calculated by either rule with the same result. With lower velocities, the rule $H = D \times R \times p \times w \times \cdot 0445$, &c., gives the lowest results: with higher velocities the rule $H = \sqrt{D \times R} \times p^2 \times w \times \mathbf{M}$, gives the lowest; in either case the rule which gives the lowest power should be taken as correct. In all ordinary cases, especially with iron-toothed wheels, the latter rule, or those in (60), will be correct. Thus, the whole of the examples in our three Tables, 10, 11, 12, are governed by that rule, and none of them by the rules for low speeds in (57). It is very seldom that the velocities met with in practice are so low that the strain may be taken as a dead load, see (66),

still it is expedient that with very low speeds, the strength should be calculated by *both* rules, and the lowest result accepted as correct.

(63.) Table 35 has been calculated by combining the rules in the manner we have explained; the points where the statical laws (57) cease, and the dynamic ones (60) prevail, are indicated by a *. This table will greatly facilitate calculations on the strength of wheels; the following examples will illustrate its application: we will take the cases in (61).

1st. To find the power of a wheel: multiply the diameter of the wheel in feet by the revolutions, find the nearest number to that in the upper line of Table 35, under which, and opposite the given pitch, will be found the horse-power per inch of width. Thus, to find the power of an iron-toothed spur-wheel 30·1 feet diameter, 15·8 revolutions, 4½ inches pitch, 16 inches wide; we have $D \times R = 30\cdot1 \times 15\cdot8 = 475\cdot5$ the nearest number to which in the table is 484, under which and opposite 4½ inches pitch, we find 19·2-horse per inch width on the face for iron-toothed wheels; in our case we have $19\cdot2 \times 16 = 307$ horse-power, which is near enough to 303·7-horse, the exact power as calculated in (61), to answer most practical purposes.

2nd. To find the width for a wheel of a given power: multiply the diameter in feet by the revolutions, find the nearest number thereto in the upper line of the table, under which and opposite the given pitch, will be found the horse-power per inch of width, dividing the given horse-power by which, we obtain the width desired. Thus, to find the proper width for a spur-mortise-wheel, 6 feet diameter, 3½ inches pitch, 25 revolutions, and 75 horse-power: $D \times R$ or $6 \times 25 = 150$, the nearest numbers to which in the table are 144 and 169, under which and opposite 3½ inches pitch, are the powers 7·3 and 7·9 horse-power per inch of width; taking 7·5 as an intermediate number, we obtain $75 \div 7\cdot5 = 10$ inches width of cog.

3rd. To find the pitch for a wheel of a given power: divide the given horse-power by the given width and we obtain the horse-power per inch which the wheel must carry; then multiply the diameter in feet by the revolutions, find the nearest number thereto in the upper line of the table, under which and opposite to the given horse-power per inch or the nearest number

thereto will be found the pitch required. Thus, to find the proper pitch for an iron-toothed spur-wheel $24\frac{1}{2}$ feet diameter, 19·5 revolutions per minute, 14 inches wide, to carry 210 horse-power. We have first $210 \div 14 = 15$ horse-power per inch; then $D \times R = 24 \cdot 5 \times 19 \cdot 5 = 478$, the nearest number to which in the table is 484, looking under which for 15-horse per inch, we find 15·1-horse per inch opposite 4 inches, the pitch required.

(64.) "*Power of Bevel-wheels.*"—The rules and tables for calculating the strength of spur-wheels, apply to bevel-wheels also with certain modifications. Bevel-wheels, as stated in (37), have a maximum and a minimum diameter, and also a maximum and minimum pitch, and in calculating their power we must not use the *reputed* sizes by which they are commonly known to practical men, but must ascertain and use the *mean* diameter, and the *mean* pitch. The best and easiest method of doing this is by drawing the pair of wheels full size, or to scale, and thus find the required dimensions by direct measurement. Thus, say, we take the case of the wheel A, Fig. 12, with 30 revolutions per minute; the maximum diameter is 3 feet 7 inches, but the minimum diameter *a* is 2 feet 8 inches only, the mean is therefore 3 feet $1\frac{1}{2}$ inch, or 3·125 feet. Again, the maximum pitch is $2\frac{1}{2}$ inches, but the minimum is less than the maximum in the ratio of the two diameters 43 and 32 inches, it is therefore $2 \cdot 5 \times 32 \div 43 = 1 \cdot 86$ inch; the mean pitch is $(2 \cdot 5 \times 1 \cdot 86) \div 2 = 2 \cdot 18$ inches. With these reduced dimensions the power of the wheel by the rule in (60) comes out $\sqrt{3 \cdot 125 \times 30} \times 2 \cdot 18^2 \times 6 \times \cdot 043 = 11 \cdot 8$ horse-power. If we had erroneously taken the reputed or maximum sizes, the power would have come out $\sqrt{3 \cdot 58 \times 30} \times 2\frac{1}{2}^2 \times 6 \times \cdot 043 =$ 16·7-horse; an error of 42 per cent. Table 12 gives the strength of bevel-mortise-wheels from cases in practice, and will also serve to illustrate further, the mode of calculation.

(65.) "*Crane-gearing.*"—The power of men in working a winch varies very much in any case with the duration of the labour, as shown by Table 4: in working a crane there are usually special circumstances which must be considered. The

time occupied in raising the load is never more than about half the reputed hours of labour, time being spent in lowering the chain for another load, &c., but where the strain on the handle is very heavy, and the labour consequently excessive, much longer intervals of rest between each load are absolutely necessary. For instance, with 24 lbs. at the handle, and a velocity of 220 feet per minute, a man can work only two hours per day, which does not mean that he would work for two hours continuously and then desist for the day, for no man would be able to do that; he would work say for two minutes and rest eight minutes, and during the day of ten hours, he would work two hours and rest eight, &c. Table 13 gives the labour of a man at a winch with varying power at the handle, &c., which will be understood from the explanation we have given, the actual day's work being ten hours in all cases, exclusive of the usual meal-times, see (15). In ordinary cases, where the time occupied in lowering, &c., is half the total time, the man working five hours and resting five, the table shows that the power at the handle is 15 lbs., which may be taken as a standard for ordinary crane-work. But in many cases only one or two loads per day have to be raised, the men come fresh to the work from other and lighter occupations, and about 30 lbs. at the handles may be obtained from able-bodied men without difficulty. Where the labour is continuous, as in working a mill, &c., 10 or 11 lbs. at the handle for ten hours per day is a fair allowance, as shown by the table.

(66.) The speed of crane-wheels is so very low, that we might suppose the strength of the teeth throughout the train might be calculated by the laws for absolute strength with a dead load (48), but the results would not accord with experience, and it will be more correct to calculate by the rules for ordinary gearing, combining those for a low speed (57) with those for ordinary velocities (60) as explained in (62).

Say that we take the gearing for a 10-ton crane to be worked by four men, and shown in outline by Fig. 46; in calculating the strength of crane-wheels it is expedient to take a maximum strength for the men, or that due to short hours of labour, say two hours per day, when by Table 13 the strain at the handles

TABLE 13.—Of the LABOUR of a MAN at a WINCH.

Duration in Hours.			Power at the Handle.	Velocity in Feet per Minute.	Foot-pounds per Minute.	Day's Work.
At Work.	At Rest.	Total.				
			lbs.			foot-pounds.
1	9	10	34	220	7467	448,020
2	8	10	24	220	5280	633,600
3	7	10	19·6	220	4311	775,980
4	6	10	17	220	3733	895,920
5	5	10	15·2	220	3340	1,002,000
6	4	10	14·3	220	3048	1,097,280
8	2	10	12	220	2640	1,267,200
10	0	10	10·73	220	2361	1,416,600

is 24 lbs., and by Table 4, ·16 horse-power, or $\cdot 16 \times 4 = \cdot 64$-horse for the four men. The velocity being 220 feet per minute is equal to 27 revolutions with a handle of 16 inches radius.

Assuming for A, B, diameters of 6 and 30 inches respectively, $1\frac{1}{4}$ inch pitch, $2\frac{3}{4}$ inches width; for C, D, 12 and 48 inches diameters, $1\frac{1}{2}$ inch pitch, $3\frac{1}{4}$ inches width; for E, F, 18 and 72 inches diameters, $1\frac{7}{8}$ inch pitch, $4\frac{1}{2}$ inches width, we can calculate the strains at the pitch-lines, the strength of the wheels, and the diameters of the shafts, throughout.

With 27 revolutions, the power of A by the rules in (60) comes out $\sqrt{\cdot 5 \times 27} \times 1\frac{1}{4}^2 \times 2\cdot 75 \times \cdot 043 = \cdot 678$-horse, which is near enough to ·64-horse, the power required. With 5·4 revolutions C becomes $\sqrt{1 \times 5\cdot 4} \times 1\frac{1}{2}^2 \times 3\cdot 25 \times \cdot 043 = \cdot 73$-horse. By the same rule the power of E would be $\sqrt{6 \times \cdot 3375} \times 1\frac{7}{8}^2 \times 4\frac{1}{2} \times \cdot 043 = \cdot 968$-horse, but we have here passed beyond the limits of the dynamic laws (62), for by the rule for very low speeds in (57), the strength comes out $6 \times \cdot 3375 \times 1\frac{7}{8} \times 4\frac{1}{2} \times \cdot 0445 = \cdot 76$-horse only, which is near enough for our purpose.

With $24 \times 4 = 96$ lbs. at the handles, the strain at the pitch-lines of A, B is $96 \times 16 \div 3 = 512$ lbs.; at C, D, $512 \times 15 \div 6 = 1280$ lbs.; at E, F, $1280 \times 24 \div 9 = 3420$ lbs., which with

a barrel 1 foot 9 inches diameter, or say 22 inches at the *centre of the chain,* becomes $3420 \times 36 \div 11 = 11{,}200$ lbs. or 5 tons on the chain, and with a running pulley as usual, doubling the power, we have 10 tons lifted by the four men.

(67.) The teeth of crane-wheels are usually of the same form as those of ordinary gearing, only that the describing circle (30) should be small so as to obtain great thickness at the root, and consequent strength: the teeth of the small pinion A, will in any case be weaker than those of its fellow B, and should be strengthened on one or both sides by shrouding up to the points (53), (54).

For crab-work, and in other cases where the wheels are subjected to rough usage, and are liable to be broken, "knuckle-gear," Fig. 44, is frequently used with advantage; they work very smoothly, are not very particular as to distance of centres or squareness of bearings, and the form is of such extreme strength that it is almost impossible to break them.

(68.) The sizes of the shafts may be found by the rules in (80); the hand-shaft G should be $\left\{\cdot 64 \div (27 \times \cdot 00135\right\} \sqrt[4]{} = 2$ inches diameter; the shaft H, $\left\{\cdot 64 \div (5\cdot 4 \times \cdot 00135\right\} \sqrt[4]{} = 3$ inches; the shaft J, $\left\{\cdot 64 \div (1\cdot 35 \times 00135\right\} \sqrt[4]{} = 4\cdot 3$ inches diameter. The main wheel F should be fixed direct on the barrel by suitable lugs and bolts so as to avoid torsional strain on the shaft K; where this cannot be done, the strain with so slow a speed as $\cdot 3375$ or one-third of a revolution per minute may be regarded as a dead load of 3420 lbs. acting with a leverage of 3 feet, namely, the radius of the wheel, and the diameter may be found by the rule:—

$$d = \sqrt[3]{} \, (w \times L \div 70)$$

In which w = the safe weight in pounds, acting with a leverage L in feet, and d = the diameter for a wrought-iron shaft. In our case we have $(3420 \times 3 \div 70) \sqrt[3]{} = 5\frac{1}{4}$ inches diameter.

(69.) "*Exceptional Cases.*"—The rules we have given, assume that the power driving a wheel is uniform in its action, and that

the work done by a wheel is uniform in its resistance. In ordinary cases these conditions are fulfilled sufficiently well, slight variations being covered by the margin of strength allowed for such contingencies in the rules. But in many cases this double-sided uniformity is not found; as an example of variable power we may take the case of a marine engine driving a screw-propeller, which, having no fly-wheel to equalize the inequalities of action of the crank, will exert a very variable power on the shaft, namely, in the case of a single cylinder, from *nothing* at the dead-points at the top and bottom of the stroke to 1·57 times the mean power of the engine, when the piston is at half-stroke, as explained more fully in (76), so that with a single engine of 100 horse-power, the strain on the gearing would vary from nothing to 157 horse-power every half-revolution, and the strength of the wheels must be adapted to the maximum power. With two cylinders at right angles there would be less inequality; the maximum power would then be 111-horse, and with three engines, the cranks equally dividing the circle = 105-horse. This may be illustrated by the gearing of the P. and O. vessel 'Pera' by Rennie, where mortise-wheels in the ratio of two to one were used for driving the screw-propeller. The large wheel was 14 feet diameter, 4 inches pitch, 48 inches wide on the face (in four sets, each 12 inches wide), with an average of 29 revolutions per minute. The gross indicated power of the engines was 1400-horse, equal by our standard in (4) to 700 nominal horse-power *mean;* the maximum power would therefore be $1 \cdot 11 \times 700 = 777$-horse, and the wheels should be adapted for that power. By the rule in (60) the calculated power comes out $\sqrt{14 \times 29} \times 16 \times 48 \times \cdot 05 = 773$ nominal horse-power.

(70.) "*Variable Work.*"—The second class of exceptional cases are those where, from the nature of the work to be done, the machinery is exposed to excessive occasional and momentary strains, to meet which the gearing must be made of extra strength; experience alone can determine the amount of extra strength required in any particular case. Thus a common 4-foot millstone grinding four bushels of wheat per hour takes 4-horse indicated, or say 3-horse nominal power (4), but the power of the stone pinion, as found necessary by long expe-

rience, is as much as 16·6-horse in some cases (146), in others 10·36-horse (142); if we take 13-horse as the average, we have $13 - 3 = 10$-horse excess of strength to resist the irregular surging action to which millstones are subjected.

Again, with rag-engines, used in paper-mills for grinding rags to pulp, a great excess of strength has been found necessary to resist excessive occasional strains to which they are subjected from the nature of the work (89). A common rag-engine takes about 6-horse nominal power, but has a pinion of 31 horse-power (151); an excess of 25 horse-power.

When a number of these extra strong wheels have to be driven by one main first-motion wheel, this latter need not be equal to the combined power of all the driven ones, for reasons given in (88). Many other classes of machinery, such as rolling and crushing mills, bone-mills, &c., &c., require wheels of extra strength, the amount varying in each particular case, and to be determined only by experience.

(71.) "*Power of Wheels with Several Pinions.*"—Our rules are adapted to the ordinary case of a *pair* of wheels, or of a wheel with one pinion; where there are two or more pinions the case is altered. So far as the strain on the teeth is concerned, a wheel will drive any number of pinions with the same ease as one, because each pinion strains a different set of teeth at the same particular moment. But obviously the wear-and-tear would be greater in proportion to the number of pinions, and the durability of the wheel would be affected in the same ratio; the strength of the arms (42) would also be affected by the number of pinions.

Say we have a 40-horse engine with a main wheel driving four pinions, each taking 10 horse-power, then each pinion should theoretically be of 10 horse-power as calculated by the rules, and of necessity the main wheel which drives the four would be by the rules of that same power,—10-horses. Here then we have a wheel apparently of 10 horse-power taking the whole power of a 40-horse engine; but the wear-and-tear would obviously be four times greater than with the normal case of one pinion.

(72.) But although this may be quite correct in principle, it

will be found expedient in practice, in order to avoid excessive wear-and-tear, to adopt somewhat larger sizes than are theoretically necessary, and this is usually done, as will be shown by (151). It is difficult to give a rule for such cases, as the question is one of expediency rather than of necessity, and we have nothing but judgment to guide us; the best rule we can give is to make the power of the main wheel inversely proportional to the *square root* of the number of pinions instead of inversely proportional to the number simply. Thus, say we have an engine of 100 horse-power with

1	2	3	4	5	6

pinions successively; now by the ordinary ratio, the wheels in these six different cases would have to be of

100	50	33	25	20	17

horse-power respectively; but admitting that the power of the wheel should be inversely as the square root of the number of pinions, we obtain

100	71	58	50	45	41

horse-power respectively.

These ratios may be used for other cases; thus in (151) we had engines of 90 horse-power, and a main wheel driving two pinions, hence by the ratio now found, that wheel should be calculated for $90 \times \cdot 71 = 64$ horse-power; it was really made 73 horse-power, but the whole case was of an exceptional character, extra strength being given throughout for reasons assigned. Again, for a wheel of 40 horse-power with five pinions, we should calculate by the rules for $40 \times \cdot 45 = 18$ horse-power instead of $40 \div 5 = 8$ horse-power; but the strength of the arms should in all cases be adapted to the actual power of the engine.

(73.) The simplest course for finding the proper width and thickness of the arms in such a complicated case is to calculate the sizes for a wheel with one pinion for 40 horse-power, the strength of the arm for which can easily be found by Table 6 or the rule in (42), and the sizes thus found may be taken as correct for a wheel carrying 40 horse-power with any number of *pinions. For instance,* say that our wheel with five pinions was

6 feet diameter, iron-and-iron toothed, and 20 revolutions per minute. We have then $D \times R$ in our case $6 \times 20 = 120$, the nearest number to which in the upper line of Table 35 is 121, looking under which, opposite say 3 inches pitch, we find 4·26-horse per inch width on the face, we therefore require a width of $40 \div 4 \cdot 26 = 9 \cdot 4$ inches. Now by Table 6 a wheel 3 inches pitch, and 9·4 inches wide, should have six arms, each $5\frac{7}{8}$ inches wide at the point and $1\frac{3}{8}$ inch thick, and this will be correct for our case, on whatever hypothesis the strength of the wheel itself may be calculated. On strict theoretical principles the wheel would be calculated for 8-horse: then looking under 121 in Table 35 opposite say $1\frac{3}{4}$ inch pitch, we find 1·45-horse per inch, and the width would be $8 \div 1 \cdot 45 = 5\frac{1}{2}$ inches, for which by Table 6 the arm should be $3\frac{1}{2}$ inches wide by $\frac{13}{16}$ inch thick. Then by the ratio for five pinions given in (43) we obtain $3 \cdot 5 \times 1 \cdot 5 = 5\frac{1}{4}$ inches width, and $\frac{13}{16} \times 1 \cdot 5 = 1\frac{1}{4}$ thickness, the sizes being rather less than those calculated in the other way.

CHAPTER III.

ON SHAFTS.

(74.) "*Strength and Stiffness of Shafts.*"—In calculating the proper diameter for a shaft it is necessary to consider, 1st, the absolute strength to resist the torsional strain; and 2nd, its stiffness or torsional elasticity. A shaft may be strong enough to resist the torsional strain upon it without twisting asunder, but it may be so elastic because of its great length as to be wholly unfit to drive any machinery in which steadiness of motion is essential; or, on the other hand, a shaft may be stiff enough to do its work because of its extreme shortness, but its strength may not be sufficient to resist the twisting strain.

It is the more necessary to consider the strength of shafts and their stiffness independently, because they follow entirely different laws: the torsional strength varies as the cube of the diameter and is independent of the length; but the torsional

stiffness varies directly as the fourth power of the diameter and inversely as the length, or $d^4 \div L$.

(75.) "*Torsional Strength.*"—Considering first, the question of the strength of shafts, irrespective of stiffness, we have the rules:

$$\mathbf{H} = d^3 \times R \div \mathbf{M}$$
$$d = \sqrt[3]{(\mathbf{M} \times \mathbf{H} \div R)}$$
$$\mathbf{M} = d^3 \times R \div \mathbf{H}$$

In which $\mathbf{H}$ = nominal horse-power by our standard (4); d = diameter in inches; and $\mathbf{M}$ = a multiplier derived from experience, and varying with the material, whether cast or wrought iron, &c., and other circumstances.

In applying these rules to practice, it will be necessary to make a distinction between steam-engine crank-shafts and ordinary ones, and to make the value of $\mathbf{M}$ much greater for the former. With common shafts the torsional strain is uniform and regular throughout the whole revolution, or nearly so, but with engine crank-shafts it is most irregular, varying from nothing at the dead-points, or when passing the centre at the top and bottom of the stroke, to a maximum when at half-stroke, when the strain exceeds greatly the mean power of the engine.

(76.) "*Steam-engine Crank-shafts.*"—Let Fig. 18 bo the outline of a steam-engine having its piston A say 1 foot diameter and 2 feet stroke, with a pressure of 20 lbs. per square inch, and let the speed be 40 revolutions per minute; omitting all consideration of friction, &c., let us see what happens in making one revolution. The piston moves twice the stroke or 4 feet, and the area of 12 inches being 113 square inches, the power exerted is $113 \times 4 \times 20 = 9040$ foot-pounds, and the strain on the crank-pin at J is $113 \times 20 = 2260$ lbs. But the weight B will not be the same as the strain on the crank-pin, although the drum C is the same diameter as the crank-path D, because while the piston moves twice the *diameter* of the crank-path, or 4 feet, the weight B ascends once the *circumference* of the same, or $3 \cdot 14 \times 2 = 6 \cdot 28$ feet, and the weight B must be less than the strain on A in the ratio of 4 to $6 \cdot 28$ or 1 to $6 \cdot 28 \div 4 = 1 \cdot 57$. The power exerted in raising the weight B varies

with every position of the crank, at F and G it is nothing, and at H and J it is greater than at B in the ratio given.

The fly-wheel E equalizes these irregularities by absorbing the extra power at H and J, and giving it out again at F and G, but evidently the crank-shaft K has a greater strain upon it than the shaft L in the ratio of 1·57 to 1, and must be made stronger to resist it, although the horse-power is the same in both cases. Thus, with 100-horse engine, the shaft K has to bear a maximum strain of 157 horse-power, and if the same value of **M** were used in both cases, its diameter would have to be calculated for that power by the rules; but the usual course is to increase the multiplier in the proper ratio.

(77.) With two engines coupled together by one shaft with cranks at a right angle, or 90° as usual, the strain is much less variable than with one crank, being 1 to 1·11 instead of 1·57, so that a single engine of 100 horse-power would require a crank-shaft equal in strength to an ordinary shaft of 157 horse-power, but two engines of the combined power of 100 horses require a shaft of 111 horse-power only.

In our case Fig. 18, the strain on the piston is 113 × 20 = 2260 lbs., and the work done in one revolution is 2260 × 4 × 40 = 361,600 foot-pounds ; the weight B is 2260 ÷ 1·57 = 1440 lbs., and the work done in raising it is 1440 × 6·28 × 40 = 361,600 foot-pounds also.

(78.) The value of **M** for cast-iron crank-shafts of steam-engines may be taken at 400, which is the old multiplier of Buchanan, and agrees very well with general practice. Table 14 gives the particulars of crank-shafts in practice, and shows that the value of **M** varies with different engines by various makers, from 398 to 576 for cast iron; the engines which gave 420 and 576 for their respective multipliers (or nearly the lowest and highest values), were by the same maker.

Wrought iron being stronger than cast iron for a torsional strain in the ratio of 14 to 9, we have 400 × 9 ÷ 14 = 260 as the value of **M** for wrought-iron crank-shafts to steam-engines; the value in Table 14 varies from 242 to 364, and were both from engines by the same maker. The shafts marked * being below $4\frac{5}{8}$ inches (79) are affected by the law of elasticity,

and in col. 3 are sized by Table 36, in which that law is considered.

With ordinary shafts the value of M will be lower than for crank-shafts, as we have seen (76): for cast iron we obtain $400 \div 1 \cdot 57 = 254$; for wrought iron, $260 \div 1 \cdot 57 = 160$.

TABLE 14.—Of CRANK-SHAFTS to STEAM-ENGINES; from Cases in Practice.

Nominal Horse-power.	Diameter in Inches. Actual.	Diameter in Inches. Calculated.	Revolutions per Minute.	Value of M.	KIND OF ENGINE.
	Wrought-iron Crank-shafts.				
100	12	$12\frac{1}{4}$	14	242	Double-cylinder.
70	$9\frac{3}{4}$	$9\frac{3}{8}$	22	291	„
60	$9\frac{1}{2}$	$9\frac{1}{4}$	$19\frac{1}{2}$	278	„
42	$8\frac{1}{4}$	8	22	294	„
30	$7\frac{1}{2}$	$6\frac{7}{8}$	$24\frac{1}{2}$	345	„
10	$4\frac{1}{2}$	$4\frac{1}{8}$*	40	364	Common high-pressure.
6	$3\frac{5}{8}$	$3\frac{5}{8}$*	40	..	„
4	$3\frac{5}{8}$	$3\frac{1}{8}$*	53	..	„
	Cast-iron Crank-shafts.				
80	13	$12\frac{1}{4}$	17·6	480	Low-pressure.
53	11	$10\frac{3}{4}$	17·0	427	„
45	11	$9\frac{3}{4}$	19·5	576	„
40	10	$9\frac{7}{16}$	19·0	475	„
36	$9\frac{1}{2}$	9	19·5	464	„
36	8	$8\frac{7}{8}$	20·5	291	High-pressure, broke.
30	9	$8\frac{3}{16}$	22·0	535	Low-pressure.
25	8	$7\frac{7}{8}$	20·5	420	„
22	7	7	25·5	398	Double-cylinder.
12	$5\frac{3}{4}$	$5\frac{1}{4}$	34·0	538	„
6	$4\frac{1}{4}$	4*	45·0	..	High-pressure.
(1)	(2)	(3)	(4)	(5)	

(79.) "*Torsional Stiffness of Shafts.*"—The rules we have given in (75) will apply without correction for large shafts, say over $4\frac{5}{8}$ inches diameter, but with smaller sizes, although the torsional strength as given by those rules may be sufficient, the elasticity or angle of torsion becomes so great that other rules, in which stiffness is considered as well as strength, become necessary. The fact that strength and stiffness follow different laws, necessitates the use of two sets of rules for shafts; with diameters above $4\frac{5}{8}$ inches shafts whose absolute torsional strength is sufficient to carry the power will be stiff enough to do ordinary work properly; but below that size, in order to obtain the proper stiffness, the diameter must be larger than necessary to give the required torsional strength.

We have stated (74) that the stiffness of a shaft varies as $d^4 \div L$, and if it were deemed necessary that all shafts, whether long or short, should be equally stiff, so that, for instance, a shaft which twists 1° with a length of 10 feet, should twist only 1° with 1000 feet, or any other length, the diameters necessary become very great. Say that we have a 2-inch shaft, 20 feet long, which is found by experience to be only just stiff enough to drive a certain delicate machine requiring great steadiness of motion, and that it is necessary to remove that machine to a distance of 200 feet, and we have to find the diameter of a shaft that will do the work with the same stiffness as before; then we find the required diameter to be $(2^4 \times 200 \div 20) \sqrt[4]{} = 3 \cdot 55$ inches.

(80.) But in most cases this extreme stiffness is not necessary, and we may admit that the angle of torsion may be allowed to increase in the same proportion as the length; in that case the length of the shaft may be neglected. Then for ordinary wrought-iron shafts we have the rules:—

$$\mathsf{H} = d^4 \times R \times \cdot 00135$$

$$d = \sqrt[4]{\left\{ \mathsf{H} \div (R \times \cdot 00135) \right\}}$$

In which d = diameter in inches, R = revolutions per minute, and H = nominal horse-power.

(81.) "*Combination of the two Rules.*"—By calculating for various sizes of shafts by these rules and by those in (75) it

will be found that with a diameter of $4\frac{5}{8}$ inches the results coincide, that is to say, both rules give the same power; with smaller sizes the rules in (80) should be used, as they give larger diameters and smaller horse-powers than the other: for larger diameters the rules in (75) should be used for the same reason. Thus with $4\frac{1}{2}$ inches diameter and say 50 revolutions, by (75) we have $4\frac{1}{2}^3 \times 50 \div 160 = 28 \cdot 5$-horse, but by (80) we obtain $4\frac{1}{2}^4 \times 50 \times \cdot 00135 = 27 \cdot 68$-horse, the rule for stiffness giving the lowest power; for $4\frac{3}{4}$ inches diameter we have $4\frac{3}{4}^3 \times 50 \div 160 = 33 \cdot 5$-horse, and $4\frac{3}{4}^4 \times 50 \times \cdot 00135 = 34 \cdot 36$-horse; here the rule for strength gives the lowest result.

(82.) A striking illustration of the necessity for considering stiffness with small shafts is given by the first example in Table 15, where we had to carry 1 horse-power with 240 revolutions of a shaft 55 feet long. By the rule in (80) we have $\left\{ 1 \div (240 \times \cdot 00135 \right\} \sqrt[4]{\ } = 1 \cdot 325$ or $1\frac{3}{8}$ inch diameter, the shaft was made $1\frac{1}{2}$ inch and did its work well: but by the rule in (75) we should have had $(160 \times 1 \div 240) \sqrt[3]{\ } = \cdot 873$ or $\frac{7}{8}$ inch diameter only, which, of course, would have been absurdly too small.

A shaft of the extraordinary length of 880 feet is at work near Bath, the diameter was $1\frac{3}{4}$ inch, and revolutions 140 per minute; it drove a simple thrashing machine, such as would usually be worked by four horses. Now, by Table 4 a horse walking in a circle for say six hours per day gives $\cdot 614$ horse-power, which being equivalent to *indicated* power by a steam-engine, would be equal to $\cdot 614 \times 2 \div 3 = \cdot 409$ nominal horse-power by our standard (4), and with four horses we have $\cdot 409 \times 4 = 1 \cdot 636$-horse. The diameter of this shaft calculated by the rule in (80) is $\left\{ 1 \cdot 636 \div (140 \times \cdot 00135 \right\} \sqrt[4]{\ } = 1 \cdot 715$ inch: the work done by a thrashing machine is of a variable character, very trying to a shaft, the feed of course being very irregular, notwithstanding which, this shaft drove the machine with perfect steadiness in spite of its extreme length, proving that the principle we have admitted in (80), that in ordinary cases the angle of torsion may be proportional to the length, is correct in practice.

(83.) Table 36 has been calculated by combining the two

rules in the manner we have illustrated; in order to adapt it to other than common wrought-iron shafts we have assumed that the stiffness and the strength of different kinds of shafts follow the same ratio, or that given by the multipliers for the latter in (78). Thus a cast-iron *crank*-shaft will carry only $160 \div 400 = \cdot 4$ of the power of an ordinary wrought-iron shaft under otherwise similar circumstances, and by multiplying the numbers in the upper line of the table by $\cdot 4$ we obtain the corresponding horse-power of cast-iron crank-shafts as given in the third line. The multipliers for ordinary cast-iron shafts and for wrought-iron crank-shafts we found to be respectively 254 and 260, or practically the same, and they are assumed to be equal in the table.

(84.) Table 36 will greatly facilitate the calculation of shafting in all ordinary cases: thus with 16 horse-power and 60 revolutions, a common wrought-iron shaft must be $3\frac{3}{4}$ inches diameter;—but if of cast iron, looking in the middle line for 16-horse we find the nearest number to be 15·9-horse, under which, the nearest number to 60 revolutions is 58, which is opposite $4\frac{1}{4}$ inches, the diameter required; a steam-engine crank-shaft for the same power and speed must be $4\frac{3}{4}$ inches in cast iron, and for wrought iron under 15·9-horse in the middle line we have 58 revolutions opposite $4\frac{1}{4}$ inches diameter, &c.

Again; a cast-iron shaft 9 inches diameter and 22 revolutions would suffice for the crank-shaft of an engine of 40 horse-power, or as an ordinary shaft for 64 horse-power; the same shaft in wrought iron would have done for the crank-shaft of an engine of 64 horse-power, or as a common shaft for 100 horse-power, &c.

Table 15 gives numerous cases of common shafts in practice, with the corresponding sizes as calculated by the rules.

(85.) "*Exceptional Cases.*"—We found in considering the strength of wheels (69) that there were many exceptional cases arising from variable power or variable resistance, necessitating some modification of the ordinary rules which assume uniformity in both; obviously shafts are subjected to the same exceptional influences, and without repeating the illustrations already given, we may proceed to show how they are affected by them.

"*Shafts for Screw-propellers.*"—The shaft from a marine engine to the screw-propeller is under exceptional conditions, as

TABLE 15.—Of COMMON SHAFTS in Practice.

Nominal Horse-power.	Revolutions per Minute.	Actual Diam.	Diam. by Table.	KIND OF WORK DONE.	Nominal Horse-power.	Revolutions per Minute.	Actual Diam.	Diam. by Table.	KIND OF WORK DONE.
		inches.					inches.		
1	240	$1\frac{1}{2}$	$1\frac{3}{8}$*	Centrifugal pump (55 ft. long).	15	13·7	7 cast	$6\frac{1}{8}$	Three-throw pumps.
1	88	$1\frac{11}{16}$	$1\frac{11}{16}$	Agricultural machinery.	16	100	$3\frac{1}{4}$	$3\frac{1}{4}$	Paper-mill, &c.
2·7	57	$2\frac{1}{8}$	$2\frac{3}{8}$	Three-throw pumps (too light).	20	234	$3\frac{3}{8}$	3	Centrifugal pump.
4	88	$2\frac{1}{2}$	$2\frac{3}{8}$	„	20	127	$3\frac{1}{2}$	$3\frac{1}{4}$	General machinery.
4	16	$3\frac{7}{8}$	$3\frac{3}{4}$	„	20	414	$2\frac{3}{4}$	$2\frac{3}{8}$	Centrifugal pump.
4	20	$4\frac{1}{4}$ cast	4	„	22	13·6	8 cast	$7\frac{1}{2}$	Three-throw pumps.
4	106	$2\frac{1}{16}$	$2\frac{1}{4}$	Envelope machinery.	24	180	$3\frac{5}{8}$	$3\frac{1}{8}$	Centrifugal pump.
6	104	$2\frac{1}{2}$	$2\frac{1}{2}$	Three-throw pumps.	24	40	$4\frac{3}{4}$	$4\frac{3}{4}$	Paper-making machine.
6	90	$2\frac{5}{8}$	$2\frac{5}{8}$	Agricultural machinery.	25	90	4	$3\frac{3}{4}$	Centrifugal pump.
6	65	$2\frac{7}{8}$	$2\frac{7}{8}$	Envelope machinery.	30	156	4	$3\frac{1}{2}$	„ (144 ft. long).
6	240	$2\frac{9}{16}$	$2\frac{1}{8}$	Centrifugal pump.	30	150	$3\frac{3}{4}$	$3\frac{1}{2}$	General machinery.
7	330	2	2	„	30	141	$3\frac{5}{8}$	$3\frac{1}{2}$	Sawing machinery.
7	217	$2\frac{1}{4}$	$2\frac{1}{4}$	„	30	125	4	$3\frac{5}{8}$	Centrifugal pump.
7	110	3	$2\frac{5}{8}$	Three-throw pump.	30	110	3	$3\frac{3}{4}$	Corn-mill, by straps (failed).
7	130	$2\frac{3}{4}$	$2\frac{1}{2}$	Agricultural machinery.	30	110	$3\frac{1}{2}$	$3\frac{3}{4}$	„ (drove pretty well).
8	105	$2\frac{7}{8}$	$2\frac{3}{4}$	„	40	148	$4\frac{1}{4}$	$3\frac{3}{4}$	Centrifugal pump.
8	110	$2\frac{7}{8}$	$2\frac{3}{4}$	Three-throw pumps.	40	152	$4\frac{1}{4}$	$3\frac{3}{4}$	„
8	150	$2\frac{3}{4}$	$2\frac{1}{2}$	Agricultural machinery.	40	300	$3\frac{5}{8}$	$3\frac{1}{8}$	„
10	80	3	$3\frac{1}{8}$	„	40	130	$4\frac{1}{2}$	$3\frac{7}{8}$	„
10	40	$3\frac{1}{2}$	$3\frac{3}{4}$	Paper-glazing rolls.	42	117	$4\frac{3}{4}$	$4\frac{1}{8}$	„
10	105	$3\frac{1}{8}$	3	Edge-runners.	42	20	9 cast	$9\frac{3}{4}$	Rag-engines, see (89), broke.
12	61	$3\frac{7}{8}$	$3\frac{1}{2}$	General machinery.	44	13·6	$8\frac{1}{2}$	8	Three-throw pumps.
12	150	$2\frac{5}{8}$	$2\frac{3}{4}$	Chaff-cutters, &c.	50	150	4	4	Sawing machinery.
14	64	$3\frac{7}{8}$	$3\frac{1}{2}$	General machinery.	60	150	$4\frac{1}{2}$	$4\frac{1}{4}$	Turbine.
15	124	$3\frac{3}{8}$	$3\frac{1}{8}$	Agricultural machinery.	60	31	$8\frac{1}{4}$	$7\frac{3}{4}$	Rag-engines, see (89).

* All these shafts are of wrought iron, except otherwise specified; none of them are engine-crank shafts.

we found to be the case with the wheels (69); having no fly-wheel to equalize the varying power of the crank, the maximum power for which the shaft must be adapted, exceeds the mean power of the engine in the ratio of 1·57 to 1 with a single engine, and 1·11 to 1 with a pair of engines coupled together in the usual way with cranks at right angles (76).

There is some difficulty in ascertaining the real nominal power of marine engines; we usually obtain only the *reputed* power of the maker, and the observed gross indicated power; the former is so uncertain (3) as to be wholly unreliable as a basis for calculating the diameter of the shaft: from the latter we may obtain the nominal horse-power by the standard we have adopted (4), and to which alone our rules apply.

Taking the case of the 'Pera' by Rennie (69), we had a pair of engines of 400 reputed horse-power, the gross indicated power was 1400-horse, equal by our standard to 700-horse nominal, which by the ratio for two engines gives a maximum power of $1·11 \times 700 = 777$ horse-power. By the ordinary rule in (75) we obtain with 58 revolutions, $(160 \times 777 \div 58) \sqrt[3]{\ } = 12\frac{7}{8}$ inches for the diameter of the shaft; but its actual size was $11\frac{3}{8}$ inches only, being $11\frac{3}{8}{}^3 \div 12\frac{7}{8}{}^3 = ·69$ or 69 per cent. only of the calculated strength.

With the 'Warrior,' we had a pair of engines by Penn, of the reputed power of 1250 horses, giving 5470 gross indicated, or 2735 nominal horse-power by our standard, and with 53 revolutions the diameter of shaft should be $(160 \times 2735 \times 1·11 \div 53) \sqrt[3]{\ } = 21$ inches, but the actual diameter was 17 inches only, being $17^3 \div 21^3 = ·53$ or 53 per cent. only of the calculated strength.

With the 'Serapis,' the engines by Humphreys and Co. were of the reputed power of 700 horses, but gave 4100 gross indicated, or 2050 nominal horse-power by our standard, and with 58 revolutions, required a shaft $(160 \times 2050 \times 1·11 \div 58) \sqrt[3]{\ } = 18\frac{1}{2}$ inches diameter: the actual diameter, 16 inches, was $16^3 \div 18\frac{1}{2}{}^3 = ·65$ or 65 per cent. of the calculated strength.

With the 'Agincourt,' a pair of engines by Maudslay, of the reputed power of 1350 horses, gave 6867 gross indicated, or 3433 nominal horse-power by our standard, and with 53 revolu-

tions, the shaft comes out $(160 \times 3433 \times 1 \cdot 11 \div 53) \sqrt[3]{} = 22\frac{5}{8}$ inches diameter: the actual size was 23 inches, so that in this case we had $23^3 \div 22\frac{5}{8}^3 = 1 \cdot 05$ or 5 per cent. *in excess.*

With the 'Simoom,' a pair of engines by James Watt and Co., of the reputed power of 400 horses, gave 1400 gross indicated, or 700 nominal horse-power, and with 62 revolutions required a shaft $(160 \times 700 \times 1 \cdot 11 \div 62) \sqrt[3]{} = 12\frac{5}{8}$ inches diameter: the actual size was $13\frac{3}{8}$ inches, being $13\frac{3}{8}^3 \div 12\frac{5}{8}^3 = 1 \cdot 14$ or 14 per cent. in *excess* of the calculated strength.

It will be observed that the practice of marine engineers is very variable, the difference arising probably from the mystification of the subject by the use of the *reputed* nominal power. In many cases propeller shafts by even the best makers are much weaker than ordinary ones: possibly the numerous failures of these shafts in practice may in some measure be due to this fact, which is much to be regretted, considering the magnitude of the interests involved. In all good rules there is a margin of extra strength allowed for contingencies, but if that limit be passed failure is eventually certain, becoming only a question of *time* (154).

(86.) "*Crank-shafts for Expansive Engines.*"—The rules for steam-engine crank-shafts are strictly adapted only to ordinary cases where the pressure on the piston is nearly uniform; but frequently high-pressure steam is used very expansively, and while the horse-power is governed by the *mean* pressure, the shaft has to bear a much higher or maximum pressure at the commencement of the stroke. Thus, say we had an engine of 20 nominal horse-power, 50 revolutions, and 45 lbs. steam, with a wrought-iron shaft: in ordinary cases such an engine would require a shaft about $4\frac{5}{8}$ in. diameter, the mean pressure being pretty nearly 45 lbs. But if this engine were required to work expansively and to cut off steam at one-third, the *mean* pressure by Table 3 would be reduced to 27 lbs., and the piston must be larger, to compensate for that reduced pressure in the ratio of 45 to 27, and the maximum pressure at the commencement being still 45 lbs., we have a strain at that moment equal to $20 \times 45 \div 27 = 33$ horse-power, the size of shaft suitable to which we find by Table 36 to be $5\frac{1}{2}$ inches diameter.

With a high-pressure, expansive, and condensing engine, the

mean pressure of 45 lb. steam cut off say at one-fourth would be 20·8, say 21 lbs. by Table 3, which, added to 13 lbs. vacuum, makes a total of 34 lbs., and the size of the piston is governed by that pressure; but at the commencement of the stroke, we have a pressure of 45 + 13 = 58 lbs., and the power exerted at that moment is 20 × 58 ÷ 34 = 34-horse, and the diameter of the shaft should be 5½ inches. The ratio of the maximum to the mean power is in both cases 1·7 to 1, and from this we obtain col. 2 in Table 16, which also shows the agreement of the rule with cases in practice.

TABLE **16**.—Of CRANK-SHAFTS to EXPANSIVE ENGINES in Practice.

Nominal Horse-power.		Revolutions.	Steam Cut off.	Diameter of Wrought-iron Shaft.		KIND OF ENGINE.
Mean.	Maximum.			Actual.	Calculated.	
9	15·3	50	¼	4½	4⅜	Condensing.
12	20·4	50	¼	4¾	4¾	,,
20	34·0	50	¼	5¼	5½	,,
45	76·5	40	¼	8¼	7¾	,,
10	17·0	60	⅓	4¼	4¼	Non-condensing.
25	42·5	40	⅓	6	6½	,,

(87.) "*Millstone Spindles, &c.*"—There are also many exceptional cases due to variable resistance. For instance, a common 4-foot millstone, grinding 4 bushels of corn per hour, and making 125 revolutions per minute, requires about 4 horse-power indicated, or 3-horse nominal, and would require for that, by the rule in (80), a wrought-iron spindle $\left\{ 3 \div (125 \times \cdot 00135 \right\} \sqrt[3]{} = 2\cdot 06$ inches diameter. But it has been found necessary in practice to increase the diameter to say 3 inches in wrought iron, or 3⅜ in cast iron, which by the table is equal to 12 or 13 horse-power. The reason for this is, that the momentum of the heavy millstone causes a surging action, which creates an excessive but irregular strain, which must be provided for by an extra strong shaft (70).

Again, a circular saw requires an extra strong spindle; thus a 4 foot 6 inch saw, making 470 revolutions, requires 10 horse-power, but has a spindle $2\frac{1}{4}$ inches diameter, which by the table is equal to 16 horse-power.

A rag-engine, used in paper-mills for grinding rags to pulp, requires about 6 horse-power, makes 160 revolutions, and has a cast-iron spindle 4 inches diameter, which per table is equal to 36 horse-power, giving an excess of $36 - 6 = 30$ horse-power, notwithstanding which they sometimes break.

(88.) "*Main Shafts.*"—Where a number of these extra strong shafts are worked by one main general one, it is not necessary in most cases to make the latter equal in power to the whole combined power of the driven shafts, for the following reason: The extraordinary strain, to meet which, extra strength is given, occurs only occasionally and irregularly, and is not likely to occur at one and the same moment in all the small shafts. For instance, if we had five pairs of 4-foot millstones in a line, worked by bevel-wheels from one long shaft, the mean power required would be $5 \times 3 = 15$ horse-power; but the power of the five spindles, as we have seen, would be $12 \times 5 = 60$ horse-power. The main shaft need not, however, be equal to 60 horse-power, for the reason just given; say we admit that two out of the five pairs of stones are taking for the moment each 12 horse-power, and the rest the normal amount of 3 horse-power each. Then the first length of shaft next the engine, &c., should be equal to $(12 \times 2) + (3 \times 3) = 33$ horse-power; and if the speed be taken at 30 revolutions per minute, the diameter by the rules or by Table 36 would be $5\frac{5}{8}$ inches in wrought iron. After passing the first pair of stones, the power would be $(12 \times 2) + (3 \times 2) = 30$-horse, and the diameter $5\frac{7}{16}$ inches; the next length would be $(12 \times 2) + (3 \times 1) = 27$ horse-power and $5\frac{1}{4}$ inches diameter, the next $12 \times 2 = 24$-horse and 5 inches diameter, and the last 12-horse and $4\frac{1}{8}$ inches diameter.

(89.) The same reasoning may be applied in fixing the sizes of long main shafts to rag-engines, saw-mills, &c., &c. In the case of rag-engines the excess of power in the shaft is very great (87), being $36 - 6 = 30$ horse-power each. This excess of strength is required when a large and compact mass of fresh

rags gets by carelessness or accident under the roll. This, however, seldom happens, and hardly ever in more than one rag-engine at one time. We may therefore admit that 30 horse-power should be added to the real net power on the shaft throughout. Rag-engines are often driven in pairs from one wheel or pair of wheels close together on the main shaft (151); say we had three pairs, requiring an engine of 36 horse-power. Then the first length next the engine must be equal to $(6 \times 6) + 30 = 66$ horse-power; and if the speed be equal to 25 revolutions per minute, the diameter by Table 36 would be $7\frac{1}{2}$ inches. After passing the first pair, the power is reduced to $(6 \times 4) + 30 = 54$ horse-power and 7 inches diameter; and the next and last is $(6 \times 2) + 30 = 42$ horse-power and $6\frac{1}{2}$ inches diameter.

That our allowance of 30-horse *extra* is not excessive may be shown by the two cases of rag-engine main shafts in Table 15. In one case a 42-horse engine driving seven rag-engines, had in the first instance a cast-iron shaft 9 inches diameter, 20 revolutions, equal by the rules in (75) to $9^3 \times 20 \div 254 = 57$-horse, which was only $57 - 42 = 15$-horse in excess of the nominal power, instead of 30 as we have allowed; the result was that the shaft broke after working a year or two, and was renewed in wrought iron, when its strength became $9^3 \times 20 \div 160 = 91$-horse, or $91 - 42 = 49$-horse excess, when of course it stood well. In the other case a 60-horse engine drove ten rag-engines, with a wrought-iron shaft $8\frac{1}{4}$ inches diameter, 31 revolutions, equal to $8\frac{1}{4}^3 \times 31 \div 160 = 109$-horse, or $109 - 60 = 49$-horse excess, being precisely the same as in the last example. In the case in (152) we had a 45-horse engine driving seven rag-engines, with a wrought-iron shaft $5\frac{1}{2}$ inches diameter and 88 revolutions, equal to $5\frac{1}{2}^3 \times 88 \div 160 = 91$-horse, or $91 - 45 = 46$-horse excess. We allowed in (152) that *two* rag-engines might be taking an excess of power at the same instant, to the extent of $62 - 14 = 48$-horse.

The principles we have adopted in fixing the sizes of main shafts in exceptional cases, are further illustrated by cases in practice in (142) (152), &c., where, however, we have taken the strength of the pinions, rather than that of the spindles as the basis of calculation.

(90.) The weight of round and square shafts of wrought iron is given by Table 17; for cast iron, multiply the tabular weights by ·91.

TABLE 17.—Of the WEIGHT of ROUND and SQUARE SHAFTS of WROUGHT IRON, 1 foot long.

Size in Inches.	Weight in Lbs.		Size in Inches.	Weight in Lbs.		Size in Inches.	Weight in Lbs.	
	Round.	Square.		Round.	Square.		Round.	Square.
⅛	·042	·053	2⅝	18·2	23·2	7¼	139	177
¼	·166	·211	2¾	20·0	25·5	7½	149	190
⅜	·372	·474	2⅞	21·9	27·9	7¾	159	203
½	·662	·843	3	23·8	30·3	8	169	216
⅝	1·03	1·32	3¼	28·0	35·6	8¼	180	229
¾	1·49	1·90	3½	32·4	41·3	8½	191	244
⅞	2·03	2·58	3¾	37·2	47·4	8¾	203	258
1	2·65	3·37	4	42·4	54·0	9	214	273
1⅛	3·35	4·27	4¼	47·8	60·9	9¼	227	288
1¼	4·14	5·27	4½	53·6	68·2	9½	239	304
1⅜	5·00	6·37	4¾	59·7	76·0	9¾	252	320
1½	5·97	7·58	5	66·2	84·3	10	265	337
1⅝	7·00	8·90	5¼	72·9	92·9	10½	292	372
1¾	8·11	10·3	5½	80·1	102·	11	320	408
1⅞	9·31	11·8	5¾	87·5	111·	11½	350	448
2	10·6	13·5	6	95·3	121·	12	381	486
2⅛	11·9	15·2	6¼	103·	132·	12½	414	527
2¼	13·4	17·1	6½	112·	142·	13	447	570
2⅜	14·9	19·0	6¾	121·	154·	13½	483	614
2½	16·5	21·1	7	130·	165·	14	519	661

ON COUPLINGS FOR SHAFTS.

(91.) "*Couplings.*"—Shafts are not often made more than 20 feet long, from the difficulty and inconvenience in making and fixing them. There are three principal forms of coupling

commonly used for round shafts—the solid coupling, with lap-joint; the flange; and the claw coupling.

"*Solid Couplings.*"—The solid coupling shown by Fig. 28 is perhaps the best of all for small shafts up to, say 4½ or 5 inches diameter; for large shafts they become clumsy and heavy. Table 18 gives the general proportions of these couplings with cast-iron boxes, and is calculated by the following rules:—

$$t = \sqrt{d} \times 1{\cdot}25$$
$$L = D \times 1{\cdot}5$$
$$l = (d \times {\cdot}75) + {\cdot}25$$

In which d = diameter of shaft; D = diameter of coupling; t = thickness of metal; L = length of coupling; and l = length of lap, all in inches.

This kind of coupling requires thoroughly good workmanship, especially in the fitting of the lap-joint; the coupling-box is secured in position by a hollow key, like Fig. 19. The angle or bevel of the joint should be about 1 inch per foot, and may be conveniently described by striking the arcs *c d* and *e f* from the point A with a radius of 6 inches, and setting off ½ inch above and below the centre line, &c., as in the figure.

TABLE 18.—Of the PROPORTIONS of SOLID HALF-LAP COUPLINGS.

Diameter of Shaft.	Thickness of Metal.	Diameter of Coupling.	Length of Coupling.	Length of Lap.
inches.	inches.	inches.	inches.	inches.
1	1	3	5¼	1
1½	1½	4½	6¾	1⅜
2	1¾	5½	8¼	1¾
2½	2	6½	9¾	2⅛
3	2⅛	7¼	10⅞	2½
3½	2⅜	8¼	12⅜	2⅞
4	2½	9	13½	3¼
4½	2⅝	9¾	14⅝	3⅝
5	2¾	10½	15¾	4

One great advantage in these couplings over most others is their safety; there are no projecting bolts, &c., as in the flange-

coupling, to catch the dress of workmen or workwomen, or to become entangled with a strap which comes off accidentally; another is, that when turned and polished, they are easily kept clean; and there are no bolts to shake loose in working, &c.

(92.) "*Flange-Couplings.*"—The flange-coupling shown by Fig. 29 is a common and useful kind for small and medium shafts, up to about 6 inches diameter. Table 19 gives the general proportions; for good work they should be turned all over, and in any case the two internal faces must be turned to fit together properly; the bolt-holes must be drilled out truly to match one another, and the bolts must be turned parallel throughout, and fit well. To keep the two shafts in line with each other, one of them should enter the opposite half-coupling, say $\frac{1}{4}$ inch in the smallest sizes, $\frac{3}{8}$ inch in medium ones, and $\frac{1}{2}$ inch in 6-inch shafts, &c. As these couplings depend for their driving power entirely on the key, that part of the work must be well and firmly done; each half should be secured by a sunk key as in Fig. 21, driven from the inside before the coupling is put together, and cut off flush, &c.; and the face should be turned true in its place after the coupling has been keyed on the shaft, otherwise, driving the key is apt to throw the coupling out of truth.

There is one objection to the use of solid and flange couplings, more especially for large shafts, namely, their rigidity, and want of accommodation in case of bad adjustment in fixing, where the perfectly straight line is not maintained, or in other cases where one of the bearings has accidentally worn down considerably more than the rest. With small shafts this is not very important, because a light shaft will spring, and so adjust itself to such irregularities; a heavy shaft may be too stiff to do so, and for that reason heavy shafts more particularly require a yielding coupling, like the following.

(93.) "*Claw-Coupling.*"—The claw-coupling shown by Figs. 30, 31, should be used for shafts above 6 inches diameter; indeed, there is no reason why it should not be used for small sizes also. When fitted together by chipping, &c., in the usual way, they are very expensive, and require good workmanship; but the expense of this fitting may be entirely avoided by

TABLE 19.—Of the PROPORTIONS of FLANGE-COUPLINGS.

Diameter of Shaft.	Diameter of Flange.	Thickness of Flange.	Diameter of Boss.	Depth at Boss.	No. of Bolts.	Diameter of Bolt.	Diameter of Circle of Bolts.
inches.	inches.	inches.	inches.	inches.		inch.	inches.
1	5	$\frac{3}{4}$	$2\frac{1}{4}$	2	3	$\frac{1}{2}$	$3\frac{1}{2}$
$1\frac{1}{2}$	$6\frac{1}{2}$	$\frac{7}{8}$	$3\frac{1}{4}$	$2\frac{1}{2}$	3	$\frac{5}{8}$	$4\frac{3}{4}$
2	8	$1\frac{1}{16}$	$4\frac{1}{4}$	3	4	$\frac{3}{4}$	6
$2\frac{1}{2}$	$9\frac{1}{2}$	$1\frac{3}{16}$	$5\frac{1}{4}$	$3\frac{1}{2}$	4	$\frac{7}{8}$	$7\frac{1}{4}$
3	11	$1\frac{3}{8}$	$6\frac{1}{4}$	4	4	1	$8\frac{1}{2}$
$3\frac{1}{2}$	$12\frac{1}{2}$	$1\frac{1}{2}$	$7\frac{1}{8}$	$4\frac{1}{2}$	4	1	$9\frac{3}{4}$
4	14	$1\frac{5}{8}$	8	5	6	1	11
$4\frac{1}{2}$	$15\frac{1}{2}$	$1\frac{7}{8}$	$8\frac{7}{8}$	$5\frac{1}{2}$	6	1	$12\frac{1}{2}$
5	17	2	$9\frac{3}{4}$	6	6	$1\frac{1}{8}$	$13\frac{1}{2}$
6	20	$2\frac{1}{4}$	$11\frac{1}{2}$	7	6	$1\frac{3}{8}$	16

casting one half *upon the other*. In that case one half is cast in sand, &c., in the usual way, and this casting being imbedded in the sand, with the wooden pattern locked into it, a mould is taken from the pattern in the usual way, and the second half is cast upon the first. A perfect fit is thus obtained without labour, and the metal is *chilled* and wears longer, so that this form of coupling is the cheapest of any. Of course, the first half must be coated with founder's blacking where the molten metal of the second half comes in contact with it, to prevent adherence. The shrinkage will be sufficient to enable the two halves to be separated when necessary, but they should be locked together, and so bored to ensure parallelism. This kind of coupling requires well securing to the shaft by good sunk keys, like Fig. 21.

BEARINGS FOR SHAFTS.

(94.) "*Plummer-blocks or Pedestals.*"—There are many modern variations in the form and general proportions of plummer-blocks; but altogether we think the old-fashioned form shown by Figs. 34 and 35, and moderate proportions, as given by Table 20, are the best for general purposes. For

special purposes these proportions may require modification; for instance, where the pressure is very heavy the length may require to be increased to obtain more area. (See 95.) Some engineers have adopted very long bearings 1½ times or even twice the diameter; such proportions are excessively heavy and expensive, and in most cases unnecessary. The principal strain on a shaft is usually a torsional one, the weight being comparatively light; an excessively wide bearing for such a case is a useless expense; it is better, therefore, to adopt moderate proportions for general purposes and to make special wide bearings for exceptional cases. In fact, the width of the bearing should not be governed by the diameter, but by the load or weight which the shaft has to carry. In cases where the upward pressure is great the hold-down bolts may require to be much stronger than in Table 20,

TABLE 20.—Of the PROPORTIONS OF PLUMMER-BLOCKS.

Diameter of Bearing.	Length of Bearing.	Height of Centre.	Length of Sole.	Centres of Hold-down Bolts.	Diameter of Bolts.	Size of Holes for Bolts.
inches.	inches.	inches.	ft. in.	ft. in.	inches.	in. in.
1½	2½	2¼	0 9	0 7	½	⅝ × 1
2	3	2¾	0 10½	0 8	⅝	¾ × 1¼
2½	3½	3⅛	1 0	0 9½	⅝	¾ × 1¼
3	4	3½	1 1½	0 10½	¾	⅞ × 1½
3½	4½	3⅞	1 3	0 11½	¾	⅞ × 1½
4	5	4¼	1 4	1 0½	⅞	1 × 1⅝
4½	5½	4⅝	1 6	1 1¾	1	1⅛ × 1¾
5	6	5⅛	1 7½	1 3	1⅛	1¼ × 2
5½	6½	5¾	1 9	1 4¼	1¼	1⅜ × 2¼
6	7	6½	1 10½	1 5	1¼	1⅜ × 2¼
7	8	7½	2 1	1 7	1½	1¾ × 2½
8	9	8½	2 5	1 11	1¾	2 × 2¾
9	10	9¼	2 6½	2 0	1½ Two	1¾ × 2½
10	11	10	2 8	2 1	1⅝ „	1⅞ × 2⅝
11	12	11	2 10	2 3	1¾ „	2 × 2¾
12	13	12	3 0	2 4	2 „	2¼ × 3

and they should be provided with double nuts, as at B, or better still, a double-headed key embracing both nuts at once may be used; but where the upward strain is *very* great the form should be altogether different, such as not to depend wholly on bolts for safety and permanence.

(95.) "*Tendency to Abrasion and Heating.*"—It has been found by experience:—1st. That when the pressure per square inch exceeds a certain amount, the oil or other lubricant is squeezed out, the contact becomes that of metal-and-metal, and the bearing cuts or abrades in spite of even constant lubrication; 2nd. That with every pressure there is a certain velocity beyond which the bearing will become heated.

Heating and abrasion are more or less connected together, for heating tends to destroy the lubricating oil, &c., and so produces abrasion with even comparatively light pressures; but it is possible for a heavy pressure to abrade yet not to heat objectionably because of the slowness of the motion, as, for instance, with the gudgeon of a water-wheel. On the other hand, the pressure may be very light, yet the bearing may heat because of the high velocity, as in the case of the spindle of an air-fan. It will therefore be convenient to consider abrasion and heating separately.

(96.) There are two or three secondary influences which have considerable effect on the tendency to heat and abrade, such as the nature of the rubbing surfaces; the kind of lubricant, and the method of its application, &c. We must assume the ordinary conditions: a wrought or cast iron journal, running in a hard gun-metal bearing, and oiled in the usual way.

It should be observed that the toe-bearings of upright shafts are more favourably situated for lubrication than ordinary bearings; with a proper oil-cup they can be maintained running *drowned* in oil, moreover, the toe is usually faced with hardened steel, which would not abrade so readily as cast or wrought iron.

(97.) "*Abrasion.*"—The tendency to abrasion is simply proportional to the mean pressure on the bearing per square inch of surface, other things, such as lubrication, &c., being the same. The direct experiments of Rennie show that with steel

TABLE 21.—Of the PRESSURE on BEARINGS in

No.	Bearing.		Assumed Mean Pressure on Piston.	Pressure on Bearing in Lbs.		Revolutions per Minute.
	Diameter.	Length.		Total.	Per square inch p.	
			lbs.			
1	14	18	..	45024*	114	23
2	9½	11	..	23500*	140	19·5
3	8¼	10¼	..	18800*	141	22
4	4¼	5½	30*	21200	576	47
5	3¼	4¼	34	10676	492	50
6	2¾	3	34	6416	500	50
7	2½	2½	34	4512	460	50
8	6	7½	20	32520	458	17½
9	5⅝	6½	20	27720	505	17½
10	3½	4¼	27	7654	327	40
11	2¼	2½	27	3051	345	60
12	2¾	4	43	10922	632	70
13	2⅛	2½	45	6411	768	40
14	2½	2¾	45	6411	600	40
15	2	2⅜	45	4564	611	45
16	3	4	60	3504	186	120
17	2⅜	3¼	60	1841	152	130
18	5	5	20	16260	414	..
19	4½	4½	20	13860	435	..
20	3½	3½	45	8550	460	..
21	3	3	45	5182	366	..
22	2¾	2¾	45	3712	312	..
23	2½	2½	45	2993	304	..
24	3	3¼	..	11000	718	..
25	8	3½	..	8930	203	15
26	1½	5¾	..	8930	660	..
27	11	..	..	20061*	211	..
28	1¾	..	..	1900*	792	125
(1)	(2)	(3)	(4)	(5)	(6)	(7)

PRACTICE, with reference to Abrasion and Heating.

Velocity of Rubbing Surface, Feet per Minute V.	Tendency to Heating. $p \times V$.	Nominal Horse-power.	KIND OF BEARING.
84·3	9630	100	Fly-wheel shaft.
48·4	6776	60	„ D. C. engine.
47·7	6726	42	„ „
52·5	30240	45	Engine crank-pin, heated.
42·5	20910	20	„ „
36·0	18000	12	„ „
32·7	15042	9	„ „
26·8	12274	80	Low-pressure, crank-pin.
26·0	13130	60	„ „
36·7	12000	25	Expansive „
35·3	12178	10	„ „
50·4	31853	..	High-pressure; „ heated badly.
22·2	17050	10	„ „ abraded slightly.
26·2	15720	10	„ „ altered; better.
23·6	14420	6	„ „
94·0	17474	8	Portable-engine „
81·0	12320	4	„ „
..	..	80	Low-pressure, link-bearing.
..	..	60	„ „
..	..	60	D. Cylinder engine „
..	..	40	„ „
..	..	30	„ „
..	..	22	„ „
..	..	..	Bow of deep-well pump.
31·5	6395	..	Head of sling to three-throw pumps.
..	..	..	Pin of ditto, lower end.
..	..	..	Toe, upright shaft.
28·6	22650	..	„ millstone spindle.
(8)	(9)	(10)	

or wrought iron on cast iron the friction increases rapidly, and abrasion ensues when the pressure exceeds $5\frac{1}{2}$ to 6 cwt. per square inch, or with cast iron on brass 7 cwt. per square inch. But the most reliable data may be obtained from cases in practice, the most instructive being those which failed or gave serious trouble by abrasion or heating. Table 21 gives a selection of such cases, the tendency to abrasion being indicated by col. 6; the heaviest pressure on a common horizontal bearing is 768 lbs. per square inch in No. 13; this did not heat, nor would it be likely to do so as we shall see presently (102), but it abraded slightly, and in making other similar engines the sizes were altered to No. 14, the pressure being thereby reduced to 600 lbs. with satisfactory results. It appears from this that with careful lubrication the pressure may be as high as say 750 lbs. per square inch, but that is about the limit, and with heavier pressures abrasion will ensue in spite of lubrication, with the surfaces we have assumed (96), namely, wrought or cast iron, on gun-metal. With toe-bearings, the toe being of hardened steel, a higher pressure may be admitted as shown by No. 28, where an ordinary spindle for a 4-foot millstone has a pressure of 792 lbs. per square inch, and with ordinary care gives no trouble by abrasion.

In horizontal bearings, we have taken the area of the surface as *half* the circumference multiplied by the length, both in inches; for toe-bearings of course the full area of the toe must be taken. Thus, with No. 3, $8\frac{1}{4}$ diameter = 26 inches circumference, hence we have an area of $(26 \div 2) \times 10 \cdot 25 = 133 \cdot 3$ square inches, and a pressure of $18800 \div 133 \cdot 3 = 141$ lbs. per square inch; again, with No. 28, $1\frac{3}{4}$ diameter of toe gives an area of $2 \cdot 4$ square inches and a pressure of $1900 \div 2 \cdot 4 =$ 792 lbs. per square inch.

(98.) From this we obtain for ordinary bearings with a maximum pressure of 750 lbs. per square inch, the rule:—

$$W = d \times l \times 10 \cdot 5$$

In which d = diameter, and l = length in inches, W = the total weight on the bearing in cwts.; the third col. in Table 22

has been calculated by this rule. Similarly, for ordinary toe-bearings to upright shafts, the rule becomes:—

$$W = d^2 \times 52 \cdot 6$$
$$\text{and} \quad w = d^2 \times 589$$

In which w = total weight in pounds, and the rest as before; cols. 2 and 3 in Table 23 have been calculated by these rules.

These rules and tables give the *maximum* pressure, beyond which abrasion will commence; in practice it is expedient to adopt a much lower pressure wherever it is possible to do so; in most cases it should not exceed 500 lbs. per square inch, or two-thirds of those given by the tables. Table 21 shows in col. 6, that in many cases the pressures are very low, the case being probably governed by other considerations, such as heating, or torsional strength.

(99.) "*Heating.*"—The tendency to heating may be represented approximately by $p \times V$, in which p = the mean pressure in pounds per square inch, and V = velocity of the rubbing surfaces in feet per minute, and thus we have obtained col. 9 in Table 21. It should be explained, that the cases marked by a * are absolute pressures from known loads or were obtained from the mean pressure shown by the indicator; all the rest are hypothetical, having been calculated in the manner illustrated in (10) with the total pressures assumed in col. 4. These latter may not have the absolute precision of the former, but have the advantage of applying without correction to new cases where the pressure must be assumed as the basis of calculation.

The cases Nos. 4 and 12 are particularly instructive as giving the limits beyond which heating will ensue; No. 4 gives trouble continually, and is endured with difficulty, $p \times V$ being 29100. No. 12 was still worse, and had to be replaced by larger sizes, $p \times V$ being in that case 31850. In most cases $p \times V$ should not exceed 20000, except perhaps with steel-faced toe-bearings where, from the hardness of the material and the favourable position for lubrication (96), we may admit 25000 No. 28 shows that 22650 answers very well in the case of mill-stone spindles.

TABLE 22.—Of the MAXIMUM LOAD on BEARINGS, with reference to Abrasion and Heating.

Bearing.		Maximum Load to avoid Abrasion.	WEIGHT ON THE BEARING, IN CWTS.														
			2	3	5	10	20	30	40	50	75	100	125	150	200	250	300
Diameter.	Length.		MAXIMUM REVOLUTIONS PER MINUTE TO AVOID HEATING.														
		cwts.															
1½	2½	39	1340	893	536	268	134	89	67								
2	3	63	1608	1072	643	322	161	107	80	64							
2½	3½	92	1876	1250	750	375	188	125	94	75	50						
3	4	126	2144	1429	857	429	214	143	107	85	57	43	34				
3½	4½	165	2412	1607	965	482	241	161	121	96	64	48	38	32			
4	5	210	2680	1786	1072	536	268	179	134	107	71	54	43	36	27		
4½	5½	260	2948	1964	1179	590	295	196	147	118	79	59	47	39	29	23	
5	6	315	..	2143	1286	643	323	214	161	129	86	64	51	43	32	26	21
5½	6½	375	..	..	1394	697	348	232	174	139	93	70	56	46	35	28	23
6	7	441	..	..	..	750	375	250	187	150	100	75	60	50	37	30	25
7	8	588	..	..	..	..	429	286	214	171	114	86	69	57	43	34	28
8	9	756	..	..	..	..	..	321	241	193	128	96	77	64	48	39	32
9	10	945	..	..	..	..	..	..	268	214	143	107	86	71	53	43	35
10	11	1155	..	..	..	..	..	..	..	236	157	118	94	79	59	47	39
11	12	1386	..	..	..	..	..	..	..	..	172	129	103	86	64	51	43
12	13	1638	..	..	..	..	..	..	..	..	..	139	111	93	70	56	46

TABLE 23.—Of the MAXIMUM LOAD on the TOE-BEARINGS for UPRIGHT SHAFTS, with reference to Abrasion and Heating.

Diameter in Inches.	Maximum Load to avoid Abrasion.		Weight on the Toe, in Cwts.														
			½	1	2	3	5	10	15	20	30	40	50	75	100	150	200
			Maximum Revolutions per Minute to avoid Heating.														
	lbs.	cwts.															
1¼	920	8	2680	1340	670	447	268										
1½	1320	12	3200	1600	800	533	320	160									
1¾	1800	16	3760	1880	940	627	376	188	125								
2	2350	21	4280	2140	1070	713	428	214	143	107							
2½	3680	33	5360	2680	1340	893	536	268	179	134	89						
3	5300	47	..	3210	1605	1070	642	321	214	160	107	80					
3½	7210	65	..	3740	1870	1247	748	374	249	187	125	93	75				
4	9420	84	..	..	2140	1427	856	428	285	214	143	107	85	56			
4½	11920	106	..	..	2410	1607	964	482	321	241	161	120	96	64	48		
5	14720	131	..	..	..	1783	1070	535	357	267	178	134	107	71	53		
5½	17810	159	..	..	..	1960	1178	588	392	294	196	147	117	78	59	39	
6	21200	189	..	..	..	..	1284	642	428	321	214	160	128	86	64	43	
7	28860	257	..	..	..	..	1498	749	500	374	250	187	150	100	75	50	37
8	37700	336	..	..	..	..	..	856	571	428	285	214	171	114	85	57	43
9	47710	426	..	..	..	..	..	963	642	481	321	241	192	128	96	64	48
10	58900	526	..	..	..	..	..	..	713	535	357	267	214	142	107	71	53
11	71270	636	..	..	..	..	..	..	785	588	392	294	235	156	118	78	59
12	84810	704	..	..	..	..	..	..	..	642	428	321	257	170	128	85	64

(100.) In finding V for toe-bearings the *mean* velocity is half that due to the extreme diameter of the toe; thus, with French millstones, the weight of a 4-foot runner would be 14 cwt., and that of the spindle, pinion, &c., say 3 cwt., giving a total of 17 cwt. or 1900 lbs., and the area of $1\frac{3}{4}$ being $2 \cdot 4$ square inches we have $p = 1900 \div 2 \cdot 4 = 792$ lbs. per square inch. Now, the circumference of $1\frac{3}{4}$ is $5 \cdot 5$ inches or $5 \cdot 5 \div 12 = \cdot 458$ foot, which with 125 revolutions gives $\cdot 458 \times 125 = 57 \cdot 2$ feet as the maximum velocity at the periphery of the toe; hence the mean velocity V, is $57 \cdot 2 \div 2 = 28 \cdot 6$ feet per minute, and $p \times V$ becomes $28 \cdot 6 \times 792 = 22650$ as in col. 9 of Table 21.

(101.) Admitting $p \times V$ to be 20000 we have for "heating," with ordinary horizontal bearings, the rules:—

$$W = 1072 \times l \div R$$
$$R = 1072 \times l \div W$$
$$l = R \times W \div 1072$$

In which W = total *maximum* weight on the bearing in cwts.; d = diameter in inches; l = length of bearing in inches; and R = *maximum* revolutions per minute.

Similarly with ordinary toe-bearings of wrought or cast iron, in hard gun-metal seats we have the rules:—

$$W = 1070 \times d \div R$$
$$R = 1070 \times d \div W$$
$$d = R \times W \div 1070$$

In which d = diameter of the toe in inches, and the rest as before. Tables 22, 23, have been calculated by these rules; they give, as stated, maximum results, and in applying them to practice a good margin should be allowed, say two-thirds or even half the tabular numbers.

(102.) It will be observed that abrasion and heating follow different laws (98) (101), a bearing which abrades may not heat, and *vice versâ;* thus in Table 21 No. 4 heated because $p \times V$ was too high, namely, 30240, but it did not abrade, p being only 576 lbs.; again No. 13 abraded, p being 768 lbs., but it did not heat, $p \times V$ being only 17050, &c.

With ordinary bearings the tendency to heating is inde-

pendent of the diameter, so that a bearing which heats with a certain load and number of revolutions would not be improved by increasing the diameter alone, because although an increase in diameter would reduce the pressure per square inch, it would also increase the velocity of the rubbing surfaces in the same proportion, and the tendency to heating would be the same as before. The best way to prevent heating is to keep down the diameter and increase the length; the power lost by friction is not increased by increasing the length, and a wide bearing will wear longer than a narrow one, and can be more easily kept cool. With very high speeds the bearings should be of great length; for instance, the spindle of an air-fan, say 1½ inch diameter, should be about 7½ inches long, or 5 to 1.

(103.) "*Distance between Bearings.*"—The number and position of bearings must be regulated by the position of the wheels or riggers on the shaft. In all cases the bearings should be as near as possible to the coupling, wheels, &c., &c. But sometimes a long shaft may have no gearing upon it for many feet, and the distance between the bearings must be fixed with reference to the stiffness of the shaft itself. We may admit that a 2-inch shaft, unloaded except by its own weight, may have bearings 10 feet apart, and allowing that the deflection may be in all cases proportional to the distance between bearings, we have the rules:—

$$L = \sqrt[3]{(d \times 16)^2}, \text{ or } L = (d = 16)^{\frac{2}{3}}$$

$$d = \sqrt[2]{L^3} \div 16, \text{ or } d = L^{\frac{3}{2}} \div 16$$

In which d = diameter of the shaft in inches, and L = length between bearings in feet. Table 24 is calculated by this rule, but it must be understood as applying only to cases where the shaft has only its own weight to carry.

TABLE 24.—Of the DISTANCE between BEARINGS of UNLOADED SHAFTS.

Diameter of the shaft in inches	1	1½	2	2½	3	4	5	6	7	8
Distance between bearings in feet	6·3	8·3	10·1	11·7	13·2	16·0	18·6	21·0	23·2	25·4

(104.) It is sometimes impracticable to get a bearing close to a wheel or rigger; in that case the shaft should be swelled, as in Fig. 24, in which the parts A and B are made of the diameter necessary by Table 36 to carry torsionally the horse-power required, and the central part, carrying the wheel C, is made of much larger diameter, as in the figure, so as to obtain the requisite stiffness. No rule for the central diameter can be given for such cases; it must be left to the judgment of the engineer.

When a wheel, &c., overhangs the end bearing of a shaft, as in Fig. 25, the neck-bearing D has not only to bear the torsional strain but also the transverse strain of the wheel, which tends to wrench the end off. In such cases the bearing D should be made of larger diameter than the rest of the shaft E, as in the figure, and between D and E the shaft may have the tapered form shown. In such cases the bearing E should be much closer than given by Table 24, say about one-third of that distance; otherwise the shaft might spring between D and E, and the requisite steadiness of the wheel, &c., would not be obtained. This is specially important in the case of a *bevel*-wheel, where a spring in the shaft would affect the position of the teeth in gear with one another.

CRANK SHAFTS FOR THREE-THROW PUMPS.

(105.) "*Cranks for 3-throw Pumps.*"—The strength of a 3-throw crank, Figs. 26, 27, must be calculated to resist two distinct strains to which it is subjected; one being the weight on the buckets due to the head of water, plus the weight of the rods, &c., which tends to break the crank transversely as a beam; and the other, the torsional strain, which tends to *twist* it asunder. In most ordinary cases the diameter required for the torsional strain is the greater of the two, but where the distance between the two bearings A, B, is greater than usual, the diameter necessary to resist the transverse strain may be the greater. The only safe course is to calculate the diameter for both strains, and adopt the one which comes out the largest.

"*Torsional Strength.*"—The size of the main bearing A may be determined by the rules already given, or by Table 36, the

TABLE 25.—Of the SIZES of THREE-THROW CRANKS for PUMPS.

Diameter of Bearings in Inches.		Transverse Strength, Reduced Safe Load in Centre, in lbs., for 1 ft. between Bearings.		NOMINAL HORSE-POWER OF CAST-IRON CRANKS.																
				1	2	3	4	5	6	8	10	12	14	16	20	25	30	35	40	50
				NOMINAL HORSE-POWER OF WROUGHT-IRON CRANKS.																
Main Bearing.	Sling Bearings.	Cast Iron.	Wrought Iron.	1·5	3·1	4·7	6·2	7·8	9·3	12	15	19	22	25	31	39	47	54	62	78
				REVOLUTIONS PER MINUTE.																
2	2·4	1844	2765	73																
2¼	2·7	2687	3937	45	90															
2½	3·0	3602	5400	30	60	90														
3	3·6	6224	9330	14	28	43	57	71												
3½	4·2	9886	14820	8	15	23	31	38	46	62										
4	4·8	14750	22120	..	9	14	18	23	27	36	46	55								
4½	5·4	21050	31490	..	..	8	11	14	17	22	28	34	39	45						
5	6·0	28820	43200	..	..	6	8	10	12	16	20	24	28	32	41	51				
5½	6·6	38350	57500	..	..	..	6	8	9	12	15	18	21	24	31	38	46	54		
6	7·2	49790	74650	..	..	..	..	6	7	9	12	14	16	19	23	29	35	41	47	59
6½	7·8	63300	94910	..	..	..	..	..	..	7	9	11	13	15	19	23	28	32	37	46
7	8·4	79060	118500	..	..	..	..	..	..	..	7	9	10	12	15	19	22	26	30	38
7½	9·0	97250	145800	..	..	..	..	..	..	..	6	7	8	10	12	15	18	21	24	30
8	9·6	118000	176900	..	..	..	..	..	..	..	..	6	7	8	10	12	15	17	20	25
9	10·8	168000	251940	..	..	..	..	..	..	..	..	..	..	6	7	9	10	12	14	18
10	12·0	230500	345600	..	..	..	..	..	..	..	..	..	..	..	5	6	8	9	10	13

end bearing may be rather smaller, as it has no torsional strain to resist. From the peculiar form of a 3-throw crank, the torsional strain on the sling-bearings C, D, E, is greater than that on the main bearing A, in the ratio of 1·73 to 1, the diameter must therefore be greater in the ratio $\sqrt[3]{1\cdot73}$ or 1·2 to 1, so that a crank whose main bearing A is 5 inches, will have its sling-bearing $5 \times 1\cdot2 = 6$ inches diameter, &c.

TABLE 26.—Of THREE-THROW CRANKS in Practice.

Horse-power, Work Done.	Gallons per Minute.	Head in Feet.*	Diameter of Bearings. Actual. Main.	Actual. Sling.	Calculated. Main.	Calculated. Sling.	Revolutions per Minute.	Material.
60·6	800	250	9	10	8·6	10·3	15	Wrought iron.
46·4	600	255	9	10	8·0	9·5	15	”
36·5	400	301	7½	9	7·45	9·0	14·1	”
27	450	200	6½	7½	6·46	7·76	16	”
25·7	314	276	7½	9	7·18	8·64	11·14	”
16·3	250	215	6	7	5·9	7·0	20	Cast iron.
13	250	170	6½	7½	6·3	7·6	13	”

* The head given includes the friction of the long delivery-pipe.

Wrought iron is by far the most trustworthy material for 3-throw cranks, especially for small and medium sizes, but the expense of forging is very great, large cranks, 6 or 7 inches in diameter, costing 1*s.* per pound from the forge, and as it is impossible to forge them very near the required form, the rough weight and the cost come out greatly in excess of that due to the exact sizes. For this reason cast iron is very generally used for cranks, and when made of the proper sizes there is not much risk of failure with fair work; very small cranks are usually made of wrought iron. Table 25 gives the proper sizes of cast and wrought iron cranks by inspection, but those sizes it should be observed are for torsional strain only. Table 26 gives the particulars of cranks in practice, with the calculated sizes for comparison; many of the cases given are of large size,

and have been at work successfully for years; other cases are given in (136).

(106.) "*Transverse Strength.*"—The transverse strength of a 3-throw crank, or its strength as a beam, may be calculated with sufficient accuracy for practical purposes by the ordinary rules for the strength of materials. Let d = the diameter of the central sling-bearing in inches, L = the length, or the distance between centres of main and end bearings in feet, and w = the safe weight or working load on the crank reduced to an equivalent central weight in pounds. Then, taking the safe working strain at $\frac{1}{10}$th of the breaking weight, we have the rules:—

For cast iron $w = d^3 \times 133 \div L$

" " $d = \sqrt[3]{(w \times L \div 133)}$

For wrought iron $w = d^3 \times 200 \div L$

" " $d = \sqrt[3]{(w \times L \div 200)}$

(107.) The application of these rules to practice will be most clearly illustrated by an example: say we have a set of 3-throw deep-well pumps, with barrels 9 inches diameter, 2 feet stroke, 15 revolutions per minute, raising the water to a total height of 330 feet including the friction of the long delivery-pipe. The capacity of a pipe or barrel in gallons per foot run will be given by the rule, $d^2 \times \cdot 341$, hence in our case with three barrels, each 2 feet stroke, we should have $9^2 \times \cdot 341 \times 6 \times 15 = 248 \cdot 6$, say 250 gallons or 2500 lbs. per minute 330 feet high, or $2500 \times 330 \div 33000 = 25$ nominal horse-power, see (2).

We must first find the diameter necessary to bear the torsional strain: by Table 25 for 25-horse with 15 revolutions, we require with cast iron, a crank $7\frac{1}{2}$ inches diameter at the main bearing A in Fig. 26, and 9 inches at the sling-bearings C, D, E. With wrought iron the diameters would be $6\frac{1}{2}$ inches at the main bearing, and $7 \cdot 8$ inches at the sling-bearings.

(108.) In calculating for the transverse strength, we must first find the weight on the crank, reducing it to an equivalent central strain; say that the pumps are fixed in a deep well with the barrels 200 feet below the crank, we should therefore have $200 \times 3 = 600$ feet of pump-rods, say $1\frac{1}{4}$ inch diameter, which

by Table 17 would weigh $4 \cdot 14 \times 600 = 2484$, say 2400 lbs., which being about equally distributed along the length of the crank would be equal to a central load of 1200 lbs. The weight of the crank itself, and of the slings, joints of the pump-rods, &c., would be about 1200 lbs., equal to 600 lbs. in the centre.

The head of water can be on *two* buckets only at one time, because one is always descending unloaded; a column of water 1 foot high giving a pressure of $\cdot 4327$ lb. per square inch, the central bucket, whose area is $63 \cdot 6$ square inches, will have a load upon it of $63 \cdot 6 \times \cdot 4327 \times 330 = 9080$ lbs. If the side sling is half-way between the centre of the crank and the bearing A or B (which is usually nearly the fact), then the load on it being also 9080 lbs. will be equal to a central load of $9080 \div 2 = 4540$ lbs. The total weight w, we now find to be $1200 + 600 + 9080 + 4540 = 15{,}420$ lbs.

We can now find the diameter necessary for the transverse strain by the rules in (106): the distance between the main bearings A and B is in our case 4 feet 6 inches, as in Fig. 26, and we have for a cast-iron crank $(15420 \times 4 \cdot 5 \div 133) \sqrt[3]{} = 8$ inches diameter of sling-bearing, whereas for the torsional strain we found (107) the diameter necessary to be 9 inches, and of course, the latter being the larger, must be adopted as the size of the crank.

For wrought iron the diameter of the central sling-bearing comes out $(15420 \times 4 \cdot 5 \div 200) \sqrt[3]{} = 7$ inches instead of $7 \cdot 8$ inches, as found necessary for the torsional strain, and here again the larger size must be used.

(109.) The third and fourth columns of Table 25 have been calculated by the rules in (106), and will enable us to ascertain easily whether the diameter we have found to be necessary for the torsional strain, will be sufficient for the transverse strain also, with the given load and length. Thus, we found in (107) that with cast iron, the sling-bearing required to be 9 inches, with which by col. 3, the safe transverse central load is 97,250 lbs. with a length of 1 foot, therefore $97250 \div 4 \cdot 5 = 21{,}610$ lbs. with a length of $4\frac{1}{2}$ feet, as in our case, and as we require 15,420 lbs. only, we have abundant transverse strength.

Again, with the wrought-iron crank, the diameter for torsional strength we found to be 7·8 inches, with which by col. 4 the safe transverse load for wrought iron is $94910 \div 4 \cdot 5 = 21{,}090$ lbs., giving an excess of strength, &c.

It will be evident that where the distance between the main bearings is great, which will happen with pumps of large diameter and low lifts, the diameter of crank necessary for the transverse strain may be greater than that required for the torsional strain, and governs the case.

(110.) "*Cranks worked by Horses.*"—The mean useful power of a horse walking in a circle, and raising water by pumps, is given by Table 4; it varies very much with the duration of the labour, but is in any case much less than the standard horse-power of 33,000 foot-pounds (1). But the maximum power exerted for a few moments occasionally under the whip, &c., may be five or six times the mean power, and if the machinery is of such a character as to resist a rapid increase in velocity, the strength of the crank and gearing must be adapted to that maximum strain. Pumps may give way a little under these circumstances, but it will be prudent to estimate for say five times the mean power given by Table 4. Thus, for six hours per day, the mean power of a horse in pumping is ·409-horse, but the crank should be adapted for $\cdot 409 \times 5 =$ say 2 horse-power: with 28 revolutions, a cast-iron crank by Table 25 must be 3 inches diameter at the main bearing, and 3·6 inches at the slings.

With more than one horse, the case would be modified; for obviously the whole number would not be exerting the maximum power at the same instant: say we have six horses, and we allow that two are exerting their maximum, and four their mean powers, we then have $(\cdot 409 \times 5 \times 2) + (\cdot 409 \times 4) = 5 \cdot 7$, or say 6 horse-power, requiring by Table 25 with twenty-seven revolutions a cast-iron crank 4 inches diameter at the main bearing, and 4·8 inches at the slings.

CHAPTER IV.

ON RIGGERS OR PULLEYS.

(111.) "*Laws of Friction, as applied to Riggers.*"—Let A, Fig. 13, be a rigger or pulley fixed so as to be incapable of turning, and T *t* weights suspended by a strap, E, which passes round the pulley, and may be caused to embrace it more or less by a small guide-pulley D. Let now the weight T be increased until the friction of the strap is overcome, and it slips on the pulley, the weight T descending.

The ratio between T and *t* varies—

1st. With the coefficient of friction of the material of the strap E sliding on the material of the pulley A.

2nd. With the *proportion* which the arc of the pulley embraced bears to the whole circumference of the pulley.

(112.) This ratio is independent of the breadth of the strap *so long as* T *and t remain the same*, but inasmuch as T and *t*, or the strain on the strap, may increase with the breadth, this must not be understood to mean that a narrow strap will drive as much as a wide one; for other things remaining the same, the strain, and therefore the driving power, varies directly and simply as the breadth.

The ratio between T and *t* is also independent of the *diameter* of the pulley, other things remaining the same; thus, for instance, a strap which slips on a pulley 1 foot diameter, with a weight of 1 cwt. at one side, and 2 cwt. at the other, would do the same on a pulley 10 feet or any other diameter, the surfaces being similar. This appears contrary to our instinctive notions, but is quite correct, as proved by experiment. But this must not be understood to mean that a small pulley will carry as much power as a large one, for obviously, if both are set in motion, making the same number of revolutions per minute, the relative speeds of strap would be proportional to the diameters, and the power would vary in the same ratio.

(113.) The laws by which the proportion of the entire circum-

ference embraced by the strap governs the ratio of the weights T t are very complicated.

Let F = the coefficient of friction.
„ L = length of circumference embraced, in feet or inches.
„ R = radius of the pulley, in the same terms as L.
„ T = the greater weight in Fig. 13, &c., or the maximum tension.
„ t = the lesser do. do. minimum tension.

Then we have the rules:—

$$T = t \times (2 \cdot 718)^{\frac{F.L}{R}}$$

$$t = T \div (2 \cdot 718)^{\frac{F.L}{R}}$$

These formulæ cannot be worked except by logarithms, and they then take the following forms:—

$$\text{Log. } T = \text{Log. } t + \left(\cdot 4343 \times \frac{F.L}{R} \right)$$

$$\text{and} \quad \text{Log. } t = \text{Log. } T - \left(\cdot 4343 \times \frac{F.L}{R} \right)$$

From Morin's experiments the coefficients of friction, or the values of F, are as follow:—

·47 for leather straps in ordinary working order on drums of wood.				
·28	do.	do.		cast iron.
·38	do.	soft and moist	do.	do.
·50 cords or ropes of hemp on pulleys or drums of wood.				

It appears from Morin's experiments that with cast-iron riggers the driving power is the same whether they are turned or not, the adhesion of the strap to the polished surface generating as much friction as with a rough surface.

(114.) If we take the case of a strap in ordinary working order on a cast-iron rigger the value of F will be ·28, and calculating for the four cases shown by Figs. 14, 15, 16, 17, in which the circumference is successively $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and wholly embraced, we find by the first formula that while $t = 1$ in all cases, T becomes

successively 1·553, 2·41, 3·77, and 5·81, as in the figures. Table 27 is calculated in this way, and gives throughout the value of T when $t = 1$, for different kinds of surface of rigger and states of strap.

When a rope is used, and it is wound more than once round the drum, the frictional power is enormous; thus, with a rough wooden drum and a rope 2·5 times round it, with $t = 1$, T is 2575, &c., &c.

(115.) "*Riggers in Motion.*"—We have so far considered the rigger as fixed; we will now apply the foregoing facts to the case of riggers in motion. The mechanical conditions of a driving rigger with half its circumference embraced by the strap are shown by Fig. 36, in which we have, as before, the rigger A and the weights T and t, as in Fig. 15, where we found them to be respectively 1 and 2·41. But in this case, the rigger A being free to turn, the weights T and t being unequal, there would be no equilibrium without an additional weight at Q, and supposing the drum J to be of the same diameter as the pulley A, it is self-evident that the sum of Q and t must be equal to T, therefore $T - t = Q$, or $2·41 - 1·0 = 1·41 = Q$.

The mechanical power transmitted by the strap, supposing Q to be raised by a rope coiled round the drum as a hoist or windlass, is the *difference* between T and t, and Q might be increased indefinitely, if we could increase T and t indefinitely in the normal proportion; there is, however, a limit to which this can be done, namely, the cohesive strength of the strap, by which the heaviest weight, T, is carried. Where leather is used we can obtain the requisite cohesive strength by increasing the width of the strap, or by making it a double or treble one, and this width must in all cases be proportional to T, and not to t or Q.

In Fig. 36, G may represent the engine-shaft, H its crank, and P the power which is equal to Q. It will be observed that the weight C, or pressure on the bearings due to the tension on the two straps, and also the maximum tension T, is much greater than the power P or weight Q.

(116.) If the weight Q had been 1·0, the maximum tension T would evidently have been $2·41 \div 1·41 = 1·71$, and the

TABLE 27.—Of the RATIO of the STRAINS on the STRAPS of DRIVING RIGGERS, &c.

Ratio of the Arc embraced by the Strap to the entire Circumference.		New Straps on Wooden Drums.		Straps in the Ordinary State on				Soft Straps on Riggers of Cast Iron.		Ropes on Wooden Drums, &c.			
				Wooden Drums.		Cast-iron Riggers.				Rough.		Polished.	
	t	T	Q	T	Q	T	Q	T	Q	T	Q	T	Q
·2	1	1·87	·87	1·80	·80	1·42	·42	1·61	·61	1·87	·87	1·51	·51
·3	1	2·57	1·57	2·43	1·43	1·69	·69	2·05	1·05	2·57	1·57	1·86	·86
·4	1	3·51	2·51	3·26	2·26	2·02	1·02	2·60	1·60	3·51	2·51	2·29	1·29
·5	1	4·81	3·81	4·38	3·38	2·41	1·41	3·30	2·30	4 81	3·81	2·82	1·82
·6	1	6·59	5·59	5·88	4·88	2·87	1·87	4·19	3·19	6·58	5·58	3·47	2·47
·7	1	9·00	8·00	7·90	6·90	3·43	2·43	5·32	4·32	9·01	8·01	4·27	3·27
·8	1	12·34	11·34	10·62	9·62	4·09	3·09	6·75	5·75	12·34	11·34	5·25	4·25
·9	1	16·90	15·90	14·27	13·27	4·87	3·87	8·57	7·57	16·90	15·90	6·46	5·46
1·0	1	23·14	22·14	19·16	18·16	5·81	4·81	10·89	9·89	23·90	22·90	7·95	6·95
1·5	1	..	..	..	..	..	..	..	..	111·31	110·31	22·42	21·42
2·0	1	..	..	..	..	..	..	..	..	535·47	534·47	63·23	62·23
2·5	1	..	..	..	..	..	..	..	..	2575·30	2574·80	178·52	177·52
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)

minimum tension t, $1\cdot0 \div 1\cdot41 = \cdot71$, and thus we obtain the strain in Fig. 37; this is the most useful form in which the question can be put, as we thus obtain the proportional maximum strain or width of strap for a unit of power at P.

With a wooden drum the friction of the surfaces is greater, and the strains for the weight Q are different, as is shown by Fig. 38. Here for $t = 1$ we find by Table 27 that T is $4\cdot38$, and hence $Q = 4\cdot38 - 1 = 3\cdot38$, as in the figure. For Q or $P = 1$, we should have $T = 4\cdot38 \div 3\cdot38 = 1\cdot29$, and $t = 1 \div 3\cdot38 = \cdot29$, &c., as in Fig. 39, so that with the same power, P, a strap $1\cdot29$ inch wide on a wooden drum would do as well as one $1\cdot71$ inch wide on a cast-iron one, as in Fig. 37.

(117.) The case of a rigger of cast iron, with more or less than half the circumference embraced by the strap, is shown by Figs. 40–43.

Thus in Fig. 40 the arc embraced being $\frac{2}{10}$, Table 27 shows (col. 7) that $T = 1\cdot42$, and t being $1\cdot0$, Q will be $1\cdot42 - 1 = \cdot42$, as in the figure. For $Q = 1$ we have $T = 1\cdot42 \div \cdot42 = 3\cdot38$, and $t = 1 \div \cdot42 = 2\cdot38$, as in Fig. 41.

With a crossed strap, as in Fig. 42, the arc embraced being $\frac{7}{10}$ths of the circumference, we have $T = 3\cdot43$ by Table 27, $t = 1$, and $Q = 2\cdot43$; and hence with $Q = 1$ we obtain $T = 3\cdot43 \div 2\cdot43 = 1\cdot41$, and $t = 1 \div 2\cdot43 = \cdot41$, &c., as in Fig. 43.

Comparing Figs. 37, 39, 41, and 43 together, it will be seen that with the same engine-power the breadth of strap would be in the ratio $1\cdot71$, $1\cdot29$, $3\cdot38$, and $1\cdot41$.

(118.) "*On the Driving Power of Riggers.*"—In applying these rules to practice, we might find the safe cohesive strength of leather, gutta-percha, or other material by direct experiment, or we might calculate it from a satisfactory case in practice. Thus let Fig. 37 represent an engine of 5 horse-power, making 50 revolutions per minute, with equal cast-iron riggers 5 feet diameter and a single leather strap 7 inches wide. Taking the *nominal* horse-power at $33000 \times 1\cdot5$, or 50,000 foot-pounds (4), and the circumference of a 5-foot rigger being $15\cdot7$ feet, P and therefore Q will be $\frac{50000 \times 5}{15\cdot7 \times 50} = 318$ lbs.; from this we find the maximum tension on the tight side of the strap, or T, to be

$318 \times 1{\cdot}71 = 544$ lbs., and the tension on the slack strap, or t, is $318 \times {\cdot}71 = 226$ lbs. The strain on the two bearings or C in Fig. 36 is $544 + 226 = 770$ lbs.; the maximum strain on the strap is $544 \div 7 = 78$ lbs. per inch wide, or say 310 lbs. per square inch of section with leather $\frac{1}{4}$ inch thick.

(119.) It will be more convenient, however, for practice to put the formula into another form, and obtain the necessary constant from experience. Let d = diameter of rigger in inches, w = width of strap in inches, R = revolutions per minute, **H** = nominal horse-power; **M** = a constant multiplier from practice, then we have the rules:—

$$w = (\mathbf{M} \times \mathbf{H}) \div (d \times \mathrm{R})$$
$$d = (\mathbf{M} \times \mathbf{H}) \div (w \times \mathrm{R})$$
$$\mathrm{R} = (\mathbf{M} \times \mathbf{H}) \div (d \times w)$$
$$\mathbf{H} = d \times w \times \mathrm{R} \div \mathbf{M}$$
$$\mathbf{M} = d \times w \times \mathrm{R} \div \mathbf{H}.$$

Thus, to find the value of **M** from the case we have just given, the rule $\mathbf{M} = d \times w \times \mathrm{R} \div \mathbf{H}$, becomes $60 \times 7 \times 50 \div 5 = 4200$. This applies only to the case in which the circumference is half embraced by the strap; if the strap had been a double one, we should have taken 2 w, &c.

TABLE 28.—Of the PROPORTIONAL WIDTH of STRAPS.

Arc embraced by the Strap.	Power. Q.	Cast-iron Riggers. T	Cast-iron Riggers. t	Wooden Drums. T	Wooden Drums. t
·2	1·0	3·38	2·38	2·25	1·25
·3	1·0	2·44	1·44	1·70	·70
·4	1·0	1·98	·98	1·44	·44
·5	1·0	1·71	·71	1·29	·29
·6	1·0	1·53	·53	1·20	·20
·7	1·0	1·41	·41	1·14	·14
·8	1·0	1·32	·32	1·10	·10
·9	1·0	1·26	·26	1·07	·07
1·0	1·0	1·21	·21	1·05	·05
(1)	(2)	(3)	(4)	(5)	(6)

We have seen (116) that the breadth of strap is in all cases proportional to T ÷ Q; taking the values of these from cols. 5 to 8 in Table 27, we obtain the numbers in Table 28, and from them we can find the proper breadth of strap in any case. Thus, we found that a 5-horse engine, with a 5-foot rigger making 50 revolutions, required a single strap 7 inches wide, when ·5 of the circumference was embraced; then, by col. 3 in Table 28, with ·2, embraced, the breadth must be $7 \times 3 \cdot 38 \div 1 \cdot 71 = 13 \cdot 8$ inches. With ·2, ·3, ·4, ·5, ·6, and ·7, the breadths come out 13·8, 10·0, 8·29, 7·0, 6·28, and 5·77 inches respectively.

(120.) Table 29 is an embodiment of these rules, &c.; and by it we can find w, P, R, or d, by inspection, for arcs between ·3 and ·7, which is sufficient for most cases in practice.

1st. To find the breadth, having the horse-power, revolutions, diameter, and arc embraced given, multiply the diameter in inches by the revolutions, and opposite the given horse-power look for the nearest number thereto, above which and opposite the given arc, will be found the breadth of strap required. Thus, for 10 horse-power, with a 5-foot rigger, only ·3 embraced and 100 revolutions, we may find the proper width of single strap, thus:—$d \times R$ is in our case $60 \times 100 = 6000$, which is found opposite 10 horse-power in col. 1, under 10 inches wide for cast-iron riggers, or 7 inches wide for a wooden drum, those widths being in both cases opposite ·3, the given arc. If the rigger had been half-embraced, the width would have been 7 and 5·3 inches respectively. With a double strap, opposite 10 horse-power in col. 1, 6000 will be found under 5 inches wide for cast-iron riggers, and 3·48 inches for wooden drum ·3 embraced, &c.

2nd. To find the power that a strap will carry; opposite the given arc look for the nearest width of strap, and under that find the number nearest to $d \times R$, opposite to which in column 1 or 2 will be found the horse-power. Say we require the power of a 6-inch strap on a pair of equal cast-iron riggers, 4 feet 3 inches in diameter with 110 revolutions. With equal riggers the arc would be ·5, and we should have $51 \times 110 = 5610$, the nearest number to which under 6 inches wide is 5600 opposite 16-horse s *a double strap*, or 8-horse as a single one.

Again, to find the power of a 4-inch crossed strap on a cast-iron rigger, 3 feet diameter, 130 revolutions, the arc being say ·6. The nearest width opposite the arc ·6 is 4·04 inches, looking under which for $36 \times 130 = 4680$ we find the nearest number to be 4670 opposite 10 horse-power with a double strap, or 5-horse with a single one, &c.

(121.) The best way of ascertaining the arc embraced in any particular case is to draw the outlines of the two riggers at the given distance of centres to scale, and measuring by steps with the compasses the length of strap in inches in contact with the rigger; dividing this by the whole circumference will give the arc required: see the examples given in column 6 of Table 30, &c. The arc required is always that on the smallest rigger of the pair.

With a pair of riggers of unequal dimensions and of the same material the strap will slip first on that rigger whose arc embraced by the strap is the smallest; with an open strap this is always the smaller of the two, except where the conditions are altered by a guide-pulley; in calculating the power of riggers, therefore, the small rigger should always be taken as a datum. With a crossed strap, whatever may be the relative diameter of the two riggers, the arc embraced will be the same for both, and of course it is unimportant which is taken as the basis of calculation.

(122.) Table 30 gives the particulars of many pairs of riggers in practice: the cases of failure are particularly instructive, and col. 9 shows that in all the cases failure might have been expected.

Thus No. 2 required a 13-inch single, or $6\frac{1}{2}$-inch double strap, and failed with an 8-inch single strap; when altered to a double strap of the same width, it drove well, see No. 3.

No. 5 failed with a 9-inch single strap to do the work for which a 14-inch single, or 7-inch double strap was required. Altered to a double strap, it drove well, see No. 6.

No. 10 required a $10\frac{1}{2}$-inch double leather strap, and a 6-inch gutta-percha one failed to do the work; larger riggers were substituted, and we then, by col. 9, required a 7-inch double strap; the 6-inch gutta-percha one did the work, but was unsatisfactory, requiring resin, &c., to increase its adhesion, see No. 11.

TABLE 29.—Of the DRIVING POWER of

Ratio of the arc embraced to whole circumference.		WIDTH OF STRAP, IN					
·3		·99	1·49	2·00	2·48	3·00	3·48
·4		·84	1·27	1·69	2·11	2·53	2·95
·5		·75	1·14	1·51	1·89	2·27	2·65
·6		·70	1·06	1·41	1·76	2·11	2·46
·7		·67	1·00	1·34	1·67	2·01	2·34
		WIDTH OF STRAP, IN INCHES, FOR CAST-					
·3		1·43	2·14	2·86	3·58	4·29	5·00
·4		1·18	1·78	2·37	2·96	3·55	4·14
·5		1·00	1·50	2·00	2·50	3·00	3·50
·6		·89	1·34	1·79	2·24	2·69	3·14
·7		·82	1·23	1·65	2·06	2·47	2·88
Nominal Horse-power.		DIAMETER OF RIGGER IN INCHES, MULTI-					
Single Leather Strap.	Double Leather Strap.						
1	2	4200	2800	2100	1680	1400	1200
2	4	8400	5600	4200	3360	2800	2400
3	6	12600	8400	6300	5040	4200	3600
4	8	16800	11200	8400	6720	5600	4800
5	10	21000	14000	10500	8400	7000	6000
6	12	25200	16800	12600	10080	8400	7200
7	14	29400	19600	14700	11760	9800	8400
8	16	33600	22400	16800	13440	11200	9600
10	20	42000	28000	21000	16800	14000	12000
12	24	..	33600	25200	20160	16800	14400
14	28	..	39200	29400	23520	19600	16800
16	32	..	..	33600	26880	22400	19200
20	40	..	..	42000	33600	28000	24000
25	50	..	..	..	42000	35000	30000
30	60	..	..	..	50400	42000	36000
35	70	..	..	..	58800	49000	42000
40	80	..	..	..	..	56000	48000
50	100	..	..	..	..	70000	60000
60	120	..	..	..	..	84000	72000
70	140	..	..	..	..	98000	84000
80	160	..	..	..	..	..	96000

Riggers or Pulleys, with Leather Straps.

Inches, for Wooden Drums.

4·00	4·47	5·00	6·00	7·00	8·00	9·00	10·00	12·00
3·78	3·80	4·22	5·06	5·91	6·75	7·60	8·44	10·13
3·03	3·41	3·79	4·55	5·30	6·06	6·82	7·58	9·09
2·81	3·17	3·54	4·23	4·94	5·64	6·34	7·05	8·46
2·67	3·01	3·44	4·01	4·68	5·35	6·02	6·69	8·03

iron Riggers, Turned or Unturned.

5·72	6·43	7·15	8·58	10·00	11·44	12·87	14·30	17·16
4·74	5·33	5·92	7·11	8·29	9·48	10·66	11·85	14·22
4·00	4·50	5·00	6·00	7·00	8·00	9·00	10·00	12·00
3·58	4·04	4·86	5·38	6·28	7·17	8·07	8·97	10·76
3·30	3·71	4·12	4·95	5·77	6·60	7·42	8·25	9·90

plied by Revolutions per Minute.

1050	934	840	700	600	525	467	420	350
2100	1868	1680	1400	1200	1050	934	840	700
3150	2802	2520	2100	1800	1575	1401	1260	1050
4200	3736	3360	2800	2400	2100	1868	1680	1400
5250	4670	4200	3500	3000	2625	2335	2100	1750
6300	5604	5040	4200	3600	3150	2802	2520	2100
7350	6538	5880	4900	4200	3675	3269	2940	2450
8400	7472	6720	5600	4800	4200	3736	3360	2800
10500	9340	8400	7000	6000	5250	4670	4200	3500
12600	11208	10080	8400	7200	6300	5604	5040	4200
14700	13076	11760	9800	8400	7350	6538	5880	4900
16800	14944	13440	11200	9600	8400	7472	6720	5600
21000	18680	16800	14000	12000	10500	9340	8400	7000
26250	23350	21000	17500	15000	13125	11675	10500	8750
31500	28000	25200	21000	18000	15750	14010	12600	10500
36750	32690	29400	24500	21000	18375	16345	14700	12250
42000	37360	33600	28000	24000	21000	18680	16800	14000
52500	46700	42000	35000	30000	26250	23350	21000	17500
63000	56040	50400	42000	36000	31500	28020	25200	21000
73500	65380	58800	49000	42000	36750	32690	29400	24500
84000	74720	67200	56000	48000	42000	37360	33600	28000

TABLE 30.—Of the DRIVING POWER of RIGGERS, from Cases in Practice.

No.	Nominal Horse-power.	Diameter of the two Riggers. Driver.	Diameter of the two Riggers. Driven.	Distance of Centres.	Arc on smallest Rigger.	Revolutions of smallest Rigger.	Width of Strap, &c.	Calculated Width of Strap.	REMARKS.
		ft. in.	ft. in.	ft. in.					
1	20	9 0	5 0	16 0	·457	65	9.D.O.	11.D.	Drove well.
2	18	12 6	3 6	20 0	·424	152	8.S.O.	13.S.	Failed; strap altered to double one; see No. 3.
3	,,	,,	,,	,,	,,	,,	8.D.O.	6½.D.	Drove well.
4	,,	6 3	1 3	19 6	·457	770	6½.G.O.	7¾.S.	Drove, but not well; same engine as No. 2.
5	12	7 0	7 0	32 0	·58	36	9.S.C.	14.S.	Failed; strap altered to double one; see No. 6.
6	,,	,,	,,	,,	,,	,,	9.D.C.	7.D.	Drove well.
7	12	10 0	3 0	16 3	·43	113	9.D.O.	5½.D.	Drove well, riggers lagged with wood.
8	12	3 0	4 9	11 0	·473	100	8.D.O.	7½.D.	Drove well.
9	12	10 0	3 0	14 0	·42	133	6.D.O.	6.D.	Drove well.
10	10	5 0	2 6	8 0	·446	80	6.G.O.	10½.D.	Failed entirely; riggers made larger, see No. 11.
11	,,	7 0	3 6	,,	·424	,,	6.G.O.	7.D.	Drove, but not well, required resin, &c.
12	,,	10 0	2 5	30 0	·472	165	6.G.O.	4½.D.	Drove well, 3 ft. 6 in. saw; same engine as No. 10.
13	10	6 0	4 5	14 6	·6	49	7.D.C.	7¼.D.	Drove well.
14	10	4 3	2 2	11 3	·6	463	4.D.C.	3¼.S.	Extra strong for 4 ft. 6 in. saw; see (87).
15	10	8 0	5 0	20 0	·476	64	9.G.O.	5½.D.	Drove well.
16	6	5 0	5 0	9 3	·5	40	6.S.O.	10.S.	Failed; strap altered to double one; see No. 17.
17	,,	,,	,,	,,	,,	,,	6.D.O.	5.D.	Drove well.
18	6	7 0	3 6	13 0	·46	106	5¾.G.O.	6½.S.	Drove well.
19	6	9 0	3 6	11 0	·416	128	5½.S.O.	5½.S.	Drove well.
20	3	1 3	2 0	12 0	·49	90	6.S.O.	9½.S.	Failed; riggers made larger; see No. 21.
21	,,	2 0	3 2	,,	·48	,,	6.S.O.	6.S.	Drove well.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	

NOTE.—S = single-leather; D = double-leather; G = gutta-percha; O = open strap; C = crossed strap.

No. 16 required a 10-inch single or 5-inch double strap, and failed to do the work with a 6-inch single strap; altered to a double strap it was satisfactory, see No. 17.

No. 20 failed with a 6-inch single leather strap to do work which by col. 9 required a $9\frac{1}{2}$-inch strap: larger riggers were substituted, a 6-inch strap was then required, and the strap of that width drove well, see No. 21.

It will be observed that in Nos. 2, 5, 16, the difficulty was overcome by converting the single strap into a double one; and in Nos. 10 and 20, by using larger riggers.

Circular saws and some other kinds of machinery require extra strength of strap, as shown by No. 14, and explained in (87).

PROPORTIONS OF RIGGERS.

(123.) "*The Proportions of Arms, &c., of Riggers.*"—The number, form, and strength of the arms, &c., of riggers is important, not only as a matter of taste, but also as affecting the safety in casting, for if the proper proportion between the strength of the arms and rim be not preserved, one or other is very likely to fly in cooling or to give way when set to work. Curved arms are the safest in this respect, their form permitting them to give way and allow the rim to contract in the casting. The particular curves to be used are arbitrary to some extent, depending on taste; a good form is shown by Fig. 32, and may be described as follows:—Draw the line A, B, and the line C, D perpendicular to it; from the point E on the line A, B, with a radius of $\frac{1}{5}$th the diameter of the rigger, draw the curve F, and from the point G in the line C, D, with a radius $\frac{1}{7}$th of the diameter, draw the curve H, and we thus obtain the centre lines of one of the arms.

(124.) Straight arms are generally preferred to curved ones, and when properly proportioned will cause no trouble from contraction in casting. The best form of section of the arm is an oval, but should not be a true ellipse, which has a heavy appearance; it should be drawn with a single curve having a radius about $\frac{5}{8}$ths of the width of the arm, and the thickness may be half the breadth, as in Fig. 33, the sharp points at A

and B being rounded off. The number of the arms is arbitrary, but in most cases four will suffice for diameters under 18 inches, six may be used up to 8 or 9 feet, and eight for larger diameters.

(125.) "*Strength of Arms.*"—Admitting that the *thickness* of arm is in all cases half the breadth, as in Fig. 33, its strength will vary as the cube of the breadth, and we have the following rules:—

$$B = \sqrt[3]{\left\{d \times w) \div (N \times 4\right\}}$$

$$b = \sqrt[3]{\left\{d \times w) \div (N \times 8\right\}}$$

In which d = the diameter of the rigger in inches: w = width of the rim in inches; B = breadth of arm at base; b = breadth of arm at the point; and N = number of arms. Table 31 has been calculated by this rule.

(126.) "*Strength of the Metal round the Eye.*"—The strength of metal in the boss varies not only with the diameter of the rigger, but also with the size of the shaft. Putting D for the diameter of the rigger in feet, d for the diameter of the shaft in inches, and t for the thickness of metal in $\frac{1}{8}$ths of an inch, we have the rule:—

$$t = D + d + 5$$

Table 32 has been calculated by this rule, which has been found to agree very well in practice. The proper size of key and form of key-boss, &c., for riggers may be found by reference to Chapter V.

(127.) The experiments of Morin show, as we have seen (113), that the power transmitted by cast-iron riggers is the same whether the rim be turned or not; but for rapid velocities it is necessary to turn the rim both inside and outside for the purpose of preserving the balance. When a rigger, &c., is out of balance, which will generally be the case as it leaves the sand, it exerts a violent shaking action on any machine to which it is attached, which may be injurious and even destructive.

(128.) "*Rounding Face for Riggers.*"—Where there is no

TABLE 31.—Of the PROPORTIONS of OVAL ARMS for RIGGERS.

Diameter of Rigger.	No. of Arms.	Width in Inches.															
		4		5		6		7		8		10		12		14	
		Breadth of Arms at Base and Point, in Inches.															
		Base.	Point.	Base.	Point.	Base.	Point.	Base.	Point.	Base.	Point.	Base.	Point.	Base.	Point.	Base.	Point.
ft. in.																	
1 0	4	1½	1¼	1⅝	1¼	1¾	1⅜	1¾	1⅜								
1 6	4	1¾	1⅜	1⅞	1½	2	1½	2	1⅝	2⅛	1¾	2¼	1⅞				
2 0	6	1⅝	1¼	1¾	1⅜	1⅞	1½	2	1⅝	2	1⅝	2¼	1¾	2⅜	1⅞		
2 6	6	1¾	1⅜	1⅞	1½	2	1½	2⅛	1¾	2¼	1¾	2⅜	1⅞	2½	2	2⅝	2⅛
3 0	6	1⅞	1½	2	1⅝	2⅛	1¾	2¼	1¾	2⅜	1⅞	2½	2	2⅝	2⅛	2¾	2¼
3 6	6	2	1⅝	2⅛	1¾	2¼	1¾	2⅜	1⅞	2½	2	2⅝	2⅛	2¾	2¼	3	2⅜
4 0	6	2	1⅝	2¼	1¾	2⅜	1⅞	2½	2	2⅝	2	2¾	2¼	2⅞	2⅜	3⅛	2½
4 6	6	2⅛	1¾	2¼	1⅞	2⅜	2	2½	2	2⅝	2⅛	2⅞	2¼	3	2⅜	3¼	2½
5 0	6	2¼	1¾	2⅜	1⅞	2½	2	2⅝	2⅛	2¾	2¼	3	2⅜	3⅛	2½	3⅜	2⅝
6 0	6	..	..	2½	2	2⅝	2⅛	2¾	2¼	2⅞	2⅜	3⅛	2½	3⅜	2⅝	3½	2¾
7 0	6	..	..	..	..	2¾	2¼	3	2⅜	3⅛	2⅜	3⅜	2⅝	3½	2¾	3¾	3
8 0	6	..	..	..	..	3	2⅜	3⅛	2⅜	3¼	2⅝	3½	2¾	3¾	2⅞	3⅞	3⅛
8 0	8	..	..	..	..	2⅝	2⅛	2¾	2¼	2⅞	2⅜	3⅛	2½	3⅜	2⅝	3½	2¾
9 0	8	..	..	..	..	..	..	2⅞	2⅜	3	2⅜	3¼	2⅝	3½	2¾	3⅝	2⅞
10 0	8	..	..	..	..	..	..	3	2⅜	3⅛	2½	3⅜	2¾	3⅝	2⅞	3¾	3

NOTE.—The thickness of the arms must be half the breadth throughout.

TABLE 32.—Of the THICKNESS of METAL ROUND the EYE of RIGGERS.

Diameter of Rigger, in Feet.	DIAMETER OF SHAFT, IN INCHES.					
	1	2	3	4	5	6
	THICKNESS ROUND EYE, IN INCHES.					
1	$\frac{7}{8}$	1	$1\frac{1}{8}$			
2	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$
3	..	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$
4	..	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$
5	..	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2
6	..	..	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{8}$
7	..	..	$1\frac{7}{8}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$
8	..	..	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$
9	..	..	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{1}{2}$

guide lever the surface should be turned rounding, say to the extent of $\frac{1}{2}$ inch per foot in width of rim; the effect of this is to cause the strap to keep in the centre notwithstanding slight errors in fixing, &c.

When a rigger is intended to run loose on a shaft, the boss should be made very deep, not less than the width of the rim of the rigger in most cases, and bushed at both ends or throughout with gun-metal.

(129.) "*Weight of Riggers.*"—The weight of riggers varies very much with the proportions adopted, and must be calculated in each case. But where the proportions are good, and the width varies with the diameter in the ratio $\sqrt[3]{D} \times 4 = b$, in which $D =$ diameter in feet and $b =$ breadth in inches, the weight may be approximately calculated by the rule:—

$$W = D^2 \times 30 \div \sqrt{D}$$

In which D is the diameter in feet and W = the weight in lbs. Table 33 is calculated by this rule, which agrees very well with

experience. For riggers of other widths and proportions of arms proper allowance must be made for such variations from the conditions assumed in the Table.

TABLE 33.—Of the WEIGHT of RIGGERS.

Diameter.		Width in Inches.	Weight.			Diameter.		Width in Inches.	Weight.		
ft.	in.		cwt.	qrs.	lbs.	ft.	in.		cwt.	qrs.	lbs.
1	0	4	0	1	6	3	0	$5\frac{3}{4}$	1	1	16
1	3	$4\frac{1}{4}$	0	1	14	3	6	$6\frac{1}{8}$	1	3	0
1	6	$4\frac{1}{2}$	0	1	27	4	0	$6\frac{3}{8}$	2	0	16
1	9	$4\frac{3}{4}$	0	2	14	4	6	$6\frac{5}{8}$	2	2	6
2	0	5	0	3	1	5	0	$6\frac{7}{8}$	3	0	0
2	3	$5\frac{1}{8}$	0	3	17	5	6	7	3	1	23
2	6	$5\frac{1}{4}$	1	0	6	6	0	$7\frac{1}{4}$	3	3	21
2	9	$5\frac{1}{2}$	1	0	25	7	0	$7\frac{5}{8}$	4	3	23

CHAPTER V.

ON THE PROPORTIONS OF KEYS FOR WHEELS AND RIGGERS.

(130.) There are five principal ways in which wheels, &c., may be secured to their places, and they are shown by Figs. 19–23. There are considerable differences in the relative efficiency and costliness of the different modes, and it must be left in most cases to the judgment of the engineer to select the one which will answer his purpose best.

Fig. 19 is a "hollow key," and is adapted only to turned shafts and bored wheels or riggers which have little work to do. The key is sunk into the boss of the rigger and is hollowed out to fit the shaft; it therefore drives only by friction, and is seldom used for anything but riggers with very light work. It has this advantage over other forms, that it allows the rigger to be easily shifted to another position on the shaft, and to

be made secure at any point without any preparation of key-bed, &c.

Fig. 20 is a "flat key," and is capable of carrying considerably more power than the hollow key; a flat is filed on the shaft to receive it, and it is sunk in the boss of the rigger or wheel. This is the common mode of fixing all riggers except those of the largest size, and may also be used for small wheels that are not very heavily loaded.

Fig. 21 is the "sunk key," and this is the most useful and trustworthy form of key for round shafts where the strain is heavy, which is always the case with powerful wheels and riggers.

(131.) "*Proportions for Sunk Keys.*"—The sizes of sunk keys, and the depth to which they should be sunk in the shaft and in the boss of the wheel or rigger, are governed by the diameter of the shaft, but are not in simple proportion to the diameter. The following empirical rules are dictated by experience:—

$$B = (D \div 4) + \cdot 125$$
$$T = (D \div 11) + \cdot 16$$
$$d = (D \div 40) + \cdot 075$$
$$d' = T - d$$

In which D = the diameter of the shaft in inches, B = the breadth of the key in inches, T = the thickness of the key, d = the depth sunk in the shaft measured at the *side* of the key (see Fig. 1), and d' = the depth sunk in the boss of the wheel, also measured at the side of the key. Table 34 is calculated by these rules, and the scales H, J, K, give the same particulars by direct measurement: to use these scales, the diameter of the shaft *on the scale* is to be taken by a rule or in the compasses, and gives the required dimension direct. Fig. 47 and Plate 10 give the proportions of sunk keys for shafts from 1 to 10 inches diameter, full size.

The breadth of the key should be equal from end to end, that is to say, it should be parallel, but the thickness must be regulated to give a uniform taper for the purpose of fitting tightly in its place. The amount of taper should be about $\frac{1}{8}$th

inch to a foot in length, and the same must be given to the key-seat in the boss of the wheel and not to the key-bed in the shaft.

TABLE 34.—Of the PROPORTIONS of SUNK KEYS for WHEELS and RIGGERS.

Diameter of Shaft, inches	1	2	3	4	5	6
Breadth of key .. inches	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{5}{8}$
Thickness of key "	·25	·34	·43	·52	·61	·71
Depth sunk in shaft "	·10	·125	·15	·175	·20	·225
Depth sunk in wheel "	·15	·215	·28	·345	·41	·485

Diameter of Shaft, inches	7	8	9	10	11	12
Breadth of key .. inches	$1\frac{7}{8}$	$2\frac{1}{8}$	$2\frac{3}{8}$	$2\frac{5}{8}$	$2\frac{7}{8}$	$3\frac{1}{8}$
Thickness of key "	·80	·89	·98	1·07	1·16	1·25
Depth sunk in shaft "	·25	·275	·30	·325	·35	·375
Depth sunk in wheel "	·55	·615	·68	·745	·81	·875

NOTE.—The depth sunk in the shaft and in the wheel is measured at the side of the key, and not at its centre, see (131) and Fig. 1; also the scales, H, J, K, Plate 4.

(132.) Fig. 22 shows the plan of hanging a wheel or rigger with four keys. It is commonly used for unturned shafts and unbored wheels. In this case there are four flat key-seats on the shaft; the keys may have the same proportions as in Table 34. The space between the shaft and the eye of the wheel allows for irregularities in casting, &c., and the exterior circumference of the rigger, or pitch-circle of the wheel, may be adjusted to run truly by regulating the thickness of the keys. The same plan is frequently adopted with square shafts, as shown by the dotted lines at B in Fig. 22.

(133.) Fig. 23 gives the mode of fixing a wheel, &c., on a square shaft with eight keys. It is undoubtedly the most secure of all, and the most reliable where the strains are very great. In fitting a wheel on this principle four temporary keys are

inserted at *a*, *b*, *c*, *d*, and by them the pitch-circle of the wheel is truly centered. The permanent keys are then accurately fitted with a taper of $\frac{1}{8}$th inch to a foot, &c., &c.

(134.) "*Key-boss for Strengthening the Key-way.*"—When a key-way is cut in the eye of a wheel or rigger, the boss is obviously weakened at that point, and a key-boss is necessary to restore the strength. The principles on which this should be done are not well understood by many practical millwrights. We sometimes see a key-boss like Fig. 10, the length of B being little greater than the width of the key A; the result is that the key-boss is useless, for the line of fracture *a c* takes the shortest course, as shown by the figure; and that course is obviously not lengthened by the boss B. The length of the course of fracture should be at least equal to the ordinary thickness of metal round the eye; in fact, it should be rather more, for it is well known that where there is a sharp corner, as at *c*, which may be regarded as an incipient crack, fracture is more likely to ensue than where no such angle is found. It is therefore advisable to make the shortest distance from *c*—say $\frac{1}{4}$ inch more than the thickness of metal round the eye. The key-bosses in the wheel and pinion (Fig. 7) are drawn on these principles, by taking a radius $\frac{1}{4}$ inch greater than the thickness of the boss and from the corner of the key, drawing the arcs *o p* and *q r*, and completing the key-boss by drawing the arc *p q* with the radius *p w*.

CHAPTER VI.

EXAMPLES OF GEARING IN PRACTICE.

(135.) The investigation of the works of good engineers is very instructive, and will enable us to test the accuracy of the rules, and to illustrate their application not only to ordinary but also to special cases. We have shown (69) (85) that exceptional cases are numerous, too much so indeed to be comprehended in any general rules; all that can be done is to show by the

analysis of some such cases, the general principles by which the engineer must be guided. Another result will be to show that the rules must not be used blindly; however perfect rules may be, there will always remain the necessity for the exercise of thoughtful judgment and practical experience.

(136.) "*Gearing to 3-throw Pumps.*"—Fig. 45 shows the arrangement of the gearing for a set of 3-throw pumps at Sutton, $11\frac{3}{4}$ inches diameter, 2 feet stroke, designed to raise 400 gallons per minute 200 feet high for low-service, and 200 gallons 400 feet for high-service; in either case the useful work done was $4000 \times 200 \div 33000 = 24 \cdot 2$ horse-power, but the engine was of 20 horse-power nominally.

The wheels A, B for low-service were 5 feet $9\frac{1}{8}$ inches, and 2 feet $3\frac{1}{8}$ inches diameters, $2\frac{3}{4}$ inches pitch, $7\frac{1}{2}$ inches wide; with 36 revolutions the power of B by the rules in (60) would be $\sqrt{2 \cdot 26 \times 36} \times 2\frac{3}{4}^2 \times 7\frac{1}{2} \times \cdot 043 = 22$-horse. The wheels C, D were 6 feet $8\frac{1}{16}$ inches, and 1 foot $4\frac{3}{16}$ inches diameters, bare 3 inches pitch, $8\frac{1}{2}$ inches wide, and the power of $D = \sqrt{1 \cdot 349 \times 36} \times 3^2 \times 8\frac{1}{2} \times \cdot 043 = 23$-horse. The pinions B, D were fixed on one barrel so as to slide endways into and out of gear with their respective wheels, care being taken to allow at S sufficient clearance so that B was fully out of gear with A before D came into gear with C.

(137.) As to the strain on the crank, it will be observed, that it is obviously a maximum at high-service, having then the same work to do as at low-service only at half the speed. The sizes may be calculated by the method explained in (105): for wrought iron, with 20 horse-power and $7 \cdot 28$ revolutions, the diameter of main bearing F comes out $\sqrt[3]{(20 \times 160 \div 7 \cdot 28)} = 7 \cdot 6$, say $7\frac{1}{2}$ inches, therefore $7 \cdot 5 \times 1 \cdot 2 = 9$ inches at the sling-bearings, G, G, G.

This gearing was tested severely by working the pumps at the quick speed, delivering 400 gallons against a total head of 260 feet, as measured by a pressure gauge on the main. The work done in that case was $4000 \times 260 \div 33000 = 31 \cdot 5$-horse; the wheels, which as we have seen were of 22 horse-power, were strained $31 \cdot 5 \div 22 = 1 \cdot 43$, or 43 per cent. above their normal

power, but seemed to bear it very well for the limited time that the experiment lasted. The speed could not be maintained with less than 47 lbs. steam in the boiler.

(138.) A similar set of pumps and gearing was used at Sevenoaks, except that the wheels A, B were 6 feet $1\frac{1}{2}$ inch and 1 foot $11\frac{3}{4}$ inches diameters, giving to the crank, speeds of 7·28 and 11·14 revolutions per minute, equal to 205 gallons against 454 feet head, or $205 \times 10 \times 454 \div 33000 = 28 \cdot 2$ horse-power, and 314 gallons against 276 feet head, or $314 \times 10 \times 276 \div 33000 = 26 \cdot 3$-horse. The calculated power of the high-service wheels C, D would be 23-horse as before (136), and that of the 3-throw crank 20-horse (137), the wheels were therefore strained $28 \cdot 2 \div 23 = 1 \cdot 23$, or 23 per cent., and the crank $28 \cdot 2 \div 20 = 1 \cdot 41$, or 41 per cent. above their normal power, but both were satisfactory in practice.

(139.) "*Catrine Water-wheel Gearing.*"—Fig. 49 gives an outline of these overshot water-wheels and their gearing as made by Fairbairn. The design was for four wheels, but two only were made; each wheel used 30 tons of water with a fall of 48 feet, equal to $2240 \times 30 \times 48 \div 33000 = 98$ horse-power gross, or 49 nominal horse-power by our standard (4); the combined power of the two water-wheels being therefore 98-horse nominal.

The wheel A was an internal one $48\frac{1}{2}$ feet diameter, $3\frac{1}{4}$ inches pitch, 15 inches wide, and $1\frac{1}{2}$ revolution per minute, equal by the rules in (60) to $\sqrt{48 \cdot 5 \times 1 \cdot 5} \times 3\frac{1}{4}^2 \times 15 \times \cdot 043 = 58$ nominal horse-power: the wheel B being similar to A we have 116 horse-power collectively. The pinions C, D were $5\frac{1}{2}$ feet diameter, and drove the central wheel E, 18 feet $3\frac{3}{4}$ inches diameter, $3\frac{1}{4}$ inches pitch, 16 inches wide, 13·3 revolutions per minute, whose power by the rule comes out $\sqrt{18 \cdot 3 \times 13 \cdot 3} \times 3\frac{1}{4}^2 \times 16 \times \cdot 043 = 113 \cdot 1$-horse nominal, or nearly the same as that of the two pinions C, D; the pinion F was $5\frac{1}{2}$ feet diameter. The mitre bevel-wheels G were 7 feet diameter, $3\frac{1}{2}$ inches pitch, 18 inches wide, but the reduced *mean* sizes (64) were 6 feet diameter, 3 inches pitch, and the calculated power becomes $\sqrt{6 \times 44} \times 3^2 \times 18 \times \cdot 043 = 113 \cdot 2$-horse nominal, or nearly *the same as the* others. These wheels G are stated to have

been designed to carry the combined power of the four water-wheels, or $98 \times 2 = 196$ horse-power nominal, for which their strength seems to be quite inadequate.

Mr. Fairbairn also states that these two water-wheels were designed to work up to a maximum power of 240 horses, or 120-horse each, which being equivalent to the *indicated* power of a steam-engine, would by our standard (4) be equal to $120 \times 2 \div 3 = 80$ nominal horse-power, or 160 for the two water-wheels.

The calculated powers 113·1, 113·2 and 116 horse-power were intermediate between 98 and 160, the actual mean and maximum powers.

(140.) "*Cotton-mill, Saltaire.*"—The gearing in this case together with the engines of the reputed or nominal power of 200 horses were by Fairbairn, and drove a cast-iron shaft 10 inches diameter making 94 revolutions per minute, which appears to be of extraordinary strength; but Fairbairn states that the *indicated* power of the engine was 600-horse, which, taken as the *gross* power, is equivalent to 300 nominal horse-power by our standard; then, by the rules in (75), taking the value of M for cast iron at 254, we obtain $(300 \times 254 \div 94)\sqrt[3]{} = 9\frac{3}{8}$ inches diameter, or $\frac{5}{8}$ inch less than the actual size. This shows the importance of using the indicated rather than the reputed nominal power of the maker as the basis of calculation whenever it is possible to do so (6); for instance, if we had taken the reputed power in this case, the diameter would have come out $(200 \times 254 \div 94)\sqrt[3]{} = 8\frac{1}{8}$ inches only.

(141.) "*Cotton-mill, Russia.*"—This is another of Fairbairn's works, in which a pair of engines of the reputed nominal power of 160 horses, but capable of working up to 600 gross indicated, was placed in the centre of a long line of cast-iron shafting which made 80 revolutions per minute. Here each shaft carried half the power or 300 gross indicated, which by our standard is equal to 150 nominal horse-power, and the calculated diameter is $(150 \times 254 \div 80)\sqrt[3]{} = 7\frac{13}{16}$ inches, or $\frac{3}{16}$ inch less than the actual size. If we had taken the reputed power we should have obtained $(80 \times 254 \div 80)\sqrt[3]{} = 6\frac{3}{8}$ inches diameter only.

The preceding cases are examples of the application of the rules to gearing working under ordinary conditions, where the

work is steady and uniform or nearly so (69); those which follow are of a different character.

(142.) "*Steam Corn-mill at Poole.*"—The arrangement of the engine and gearing in this mill is shown by Fig. 50; there were eight pairs of 4-feet French stones, each grinding about 4 bushels of wheat per hour, equal to 4-horse indicated, or 3-horse nominal (4) and $3 \times 8 = 24$-horse total for grinding. The shaft K drove the dressing machine, bolter, and sack-tackle; to dress a bushel of flour requires about $\frac{1}{4}$th of an indicated horse-power, and as we have 32 bushels per hour we have $32 \div 4 = 8$-horse indicated or $8 \times 2 \div 3 = 5 \cdot 3$ horse-power nominal for dressing; adding, say $1 \cdot 7$-horse for the bolter and sack-tackle, and we find that the shaft K carries 7 horse-power.

As explained in (70) the surging action of millstones, due to irregular momentum, necessitates extra strong gearing; in our case the mortise bevel-wheels were 5 feet 5 inches, and 1 foot $4\frac{9}{16}$ inches diameters, 2 inches pitch, $4\frac{3}{4}$ inches wide; but the reduced *mean* diameter (64) of the pinion was $1 \cdot 3$ foot, and its mean pitch $1 \cdot 85$ inch, hence the calculated power by the rules in (60) with 125 revolutions becomes $\sqrt{1 \cdot 3 \times 125} \times 1 \cdot 85^2 \times 4 \cdot 75 \times \cdot 05 = 10 \cdot 36$ horse-power, so that we have an excess of $10 \cdot 36 - 3 = 7 \cdot 36$ horse-power. Say we take the *excess* at 7-horse, and admit that 3 out of the 8 pairs (88) are taking $7 + 3 = 10$ horse-power, and the rest the normal amount of 3-horse. Then, the cast-iron shaft J, as we have seen, carries 7-horse, or the same as K, and with $31 \cdot 8$ revolutions, should be 4 inches diameter by Table 36. The shaft H carries $7 + 3 + 7 = 17$-horse $= 5\frac{1}{4}$ inches diameter; the shaft $G = 17 + 3 + 7 = 27$-horse, and 6 inches diameter; the shaft $F = 27 + 3 + 7 = 37$-horse, and $6\frac{1}{2}$ inches diameter; the shaft $E = 37 + 3 = 40$-horse, and the diameter by the rule in (75) is $(254 \times 40 \div 31 \cdot 8)\sqrt[3]{} = 6\frac{7}{8}$ inches; the shaft $D = 40 + 3 = 43$-horse, and $(254 \times 43 \div 31 \cdot 8)\sqrt[3]{} = 7$ inches diameter; the shaft $C = 43 + 3 = 46$-horse, and $(254 \times 46 \div 31 \cdot 8)\sqrt[3]{} = 7\frac{3}{16}$ inches diameter; the shaft $B = 46 + 3 = 49$-horse, and $(254 \times 49 \div 31 \cdot 8)\sqrt[3]{} = 7\frac{5}{16}$ inches diameter; and, finally, the shaft $A = 49 + 3 = 52$-horse, and $(254 \times 52 \div 31 \cdot 8)\sqrt[3]{} = 7\frac{1}{2}$ inches diameter.

(143.) The shafting, which was of turned cast iron, was nearly of the calculated sizes, except toward the remote end, where it was considerably stronger, notwithstanding which, it was not at all satisfactory; its elasticity was such that the last two or three pairs of stones drove so unsteadily that eventually the shaft K was removed to the position L, which had the desired effect. It will be observed that the combined calculated strength of the bevel-wheels was $10 \cdot 36 \times 9 = 93$-horse, the maximum power of the shaft = 52-horse, and the maximum useful work done in grinding and dressing, &c. $(3 \times 8) + 7 = 31$ horse-power; usually, however, one pair of stones would be up to be dressed, and the power would then be 28-horse. The engine, nominally of 25 horse-power, did the work satisfactorily; but obviously, if that reputed nominal power had been taken as a basis in calculating the sizes of the shafting the result would have been an utter failure.

(144.) "*Water Corn-mill, Bishops-Stortford.*"—The outline of this mill is given by Fig. 51; it consisted of a common breast wheel, with 7 feet 3 inches fall, which drove four pairs of 4-feet French stones, and the usual dressing machine, bolter, sack-tackle, &c. Each pair of stones grinding 4 bushels of wheat per hour, required 4-horse indicated or 3-horse nominal power, we had therefore $3 \times 4 = 12$ horse-power for grinding; to dress 16 bushels of flour requires 4-horse indicated or $4 \times 2 \div 3 = 2 \cdot 7$-horse nominal, and adding say $1 \cdot 3$-horse for bolter, &c., gives a total of 16 nominal horse-power.

The wheel A was an internal one, iron-and-iron, 8 feet $3\frac{3}{4}$ inches diameter, its pinion B, 3 feet $6\frac{7}{8}$ inches diameter, $2\frac{3}{4}$ inches pitch, 6 inches wide, and the power by the rule in (60) is $\sqrt{3 \cdot 7 \times 11 \cdot 52} \times 2\frac{3}{4}^2 \times 6 \times \cdot 043 = 12 \cdot 74$-horse, or 20 per cent. less than the strength required, notwithstanding which it appears to have been satisfactory in practice (147).

The mortise bevel-wheels C, D were 7 feet $5\frac{5}{16}$ inches, and 3 feet $3\frac{3}{8}$ inches diameters, $2\frac{3}{4}$ inches pitch, $5\frac{3}{4}$ inches wide. The reduced *mean* diameter (64) of C was 7 feet, and pitch $2 \cdot 58$ inches, hence the power is $\sqrt{7 \times 11 \cdot 52} \times 2 \cdot 58^2 \times 5 \cdot 75 \times \cdot 05 = 17 \cdot 19$-horse.

The spur-mortise-wheels E, F were 7 feet $1\frac{5}{16}$ inch, and 1 foot $5\frac{5}{8}$ inches diameters, 2 inches pitch, $4\frac{3}{4}$ inches wide, and the calculated power of the stone pinion F is $\sqrt{1 \cdot 47 \times 125} \times 2^2 \times 4 \cdot 75 \times \cdot 05 = 12 \cdot 88$-horse: as there were four such pinions, their collective power is $51 \cdot 52$-horse.

The mortise bevel-wheels G, H drove the dressing and sack-tackle, and were 5 feet $6\frac{7}{8}$ inches, and $11\frac{3}{4}$ inches diameters, $1\frac{3}{4}$ inch pitch, 4 inches wide; the reduced (64) mean diameter of G is 5 feet 3 inches, and pitch $1 \cdot 66$ inch, hence the power is $\sqrt{5 \cdot 25 \times 26 \cdot 12} \times 1 \cdot 66^2 \times 4 \times \cdot 05 = 6 \cdot 45$-horse.

(145.) Now, the pinion D has to drive wheels of the collective power of $51 \cdot 52 + 6 \cdot 45 = 58$-horse, and yet its calculated power as we have seen is only $17 \cdot 19$-horse, or 7 per cent. only in excess of the *mean* work done, namely, 16-horse; the case is exceptional, and is governed by the strength which the stone pinion requires to bear occasional shocks as explained in (70) (87). But this extra strain would in all probability occur with one pair of stones only, at the same moment, and the other three pairs would at that time be taking the normal amount of 3-horse, so that the combined strain on the large wheel E would be $9 + 13 = 22$-horse, instead of $51 \cdot 52$-horse. Even this reduced strain does not appear to be transmitted to the general gearing, for adding 4-horse for dressing, the strain becomes $22 + 4 = 26$-horse, while the strength of the wheels C, D is $17 \cdot 19$, and of A, B, $12 \cdot 74$-horse. The probability is, that the four stone pinions and the wheel E, equalize the strain among themselves, so that when an occasional momentary strain occurs to one of them, it is absorbed by the others, and thus the *mean* strain alone is borne by the general gearing. We have in our case, four stones, weighing 14 cwt. each, making 125 revolutions; these would in effect become so many fly-wheels, and would act as such in absorbing and equalizing the extraordinary strain to which one or more might be subjected for a few seconds.

This principle of mutual giving-and-taking, and of equalization as the result, applies only to cases where there is a very close connection, as in the one we have considered, where several pinions are geared into one main wheel: where each pinion has

its own driving wheel, with a length of shafting between each, as in the last case (142), the principle of equalization does not seem to apply.

(146.) "*Water Corn-mill, Risely.*"—Fig. 52 gives an outline of the gearing of this mill, which is somewhat similar to the last example, and will illustrate the same general principles of construction. In this case there was a breast wheel 14 feet diameter, with a fall of $5\frac{1}{2}$ feet, which drove four pairs of 4-feet French stones, and the usual dressing tackle, &c. The useful work done is the same as in the last example (144), namely, 12-horse for grinding, and 4-horse for dressing, or 16-horse total.

The wheel K was a bevel-mortise, 10 feet $3\frac{1}{8}$ inches diameter, and its pinion L = 3 feet $1\frac{1}{4}$ inch diameter, $2\frac{3}{4}$ inches pitch, $6\frac{1}{8}$ inches wide; the reduced (64) mean diameter of K was 9 feet 9 inches, and pitch $2\frac{5}{8}$ inches, hence the calculated power by the rules in (60) is $\sqrt{9 \cdot 75 \times 5 \cdot 65} \times 2\frac{5}{8}^2 \times 6 \cdot 125 \times \cdot 05 = 15 \cdot 66$, or very nearly the mean power required.

The spur-mortise-wheels M, N were 9 feet 7 inches, and 1 foot $5\frac{1}{4}$ inches diameters, $2\frac{1}{8}$ inches pitch, $5\frac{1}{2}$ inches wide, hence the calculated power of the stone pinion N is $\sqrt{1 \cdot 44 \times 125} \times 2\frac{1}{8}^2 \times 5 \cdot 5 \times \cdot 05 = 16 \cdot 6$-horse, which is $16 \cdot 6 - 3 = 13 \cdot 6$-horse in excess of the mean power required to drive the stones. In this case, as in the last, the strength of the stone pinion is adapted to the *maximum* strain to which it is exposed, but the strength of the rest of the gearing is adapted to the *mean* strain, or useful work done, as explained more fully in (145). The water-wheel was a wooden one, with thirty-six open buckets, 14 inches deep, and the speed of the bucket was $4 \cdot 14$ feet per second.

(147.) "*Water-wheel and Rag-engines, Dartford.*"—As stated in (89) the rag-engines used in paper-mills to grind the rags to pulp are exposed to excessive shocks, and require gearing of extraordinary strength compared to the mean power required to drive them, and it will be instructive to see how far the general gearing driving several rag-engines is affected thereby.

Fig. 56 gives an outline of the gearing for four rag-engines, driven by a common breast wheel 18 feet diameter, the speed of

bucket being 4·37 feet per second. The *mean* power required by an ordinary rag-engine being 6-horse nominal, we have 24 horse-power in our case.

The iron-toothed first-motion wheels A, B were 10 feet 6 inches, and 4 feet $4\frac{1}{2}$ inches diameters, $2\frac{1}{2}$ inches pitch, $8\frac{3}{4}$ inches wide, the power of A, calculated by the rules in (60) is $\sqrt{10 \cdot 5 \times 4 \cdot 37} \times 2\frac{1}{2}^2 \times 8 \cdot 75 \times \cdot 043 = 16$-horse; but the actual power as we have seen was 24-horse, or 50 per cent. in excess of the calculated power; these wheels were too weak, and as the result gave much trouble, on two occasions a tooth broke; eventually stronger wheels were substituted. In (138) we had a case where wheels were strained say one-sixth of their time (when working on high-service) 23 per cent. above their calculated power; and in (137) for a few hours 43 per cent., which seems to be the utmost limit. With 50 per cent. as in our present case, they will sooner or later break down. The 60-horse wheels in Table 10 broke after a year or two with a strain $60 \div 51 \cdot 1 = 1 \cdot 17$ or 17 per cent. only in excess of their calculated power, but the work done, namely, driving rag-engines, was of an exceptional character (70) (151), for which sufficient allowance was not made (89).

The mortise-wheels C, D were 9 feet $7\frac{9}{16}$ inches, and 4 feet $4\frac{1}{2}$ inches diameters, $2\frac{3}{4}$ inches pitch, 6 inches wide; hence the calculated power of C is $\sqrt{9 \cdot 6 \times 10 \cdot 46} \times 2\frac{3}{4}^2 \times 6 \times \cdot 05 = 22 \cdot 74$-horse, which is pretty nearly the mean power required; these wheels gave no trouble.

The mortise-wheel E was 11 feet $1\frac{5}{8}$ inch diameter, $2\frac{1}{2}$ inches pitch, 5 inches wide, and drove two pinions F, 1 foot $7\frac{7}{8}$ inches diameter, the calculated power of which is $\sqrt{1 \cdot 65 \times 160} \times 2\frac{1}{2}^2 \times 5 \times \cdot 05 = 25 \cdot 4$-horse, and as with the two wheels E, E there were four such pinions, we have apparently a strength of gearing equal to $25 \cdot 4 \times 4 = 101 \cdot 6$ horse-power; but, as we have seen, the pinion D which drives the whole satisfactorily, was of only 22·74 horse-power.

As stated in (89) the extraordinary strain, to meet which such great extra strength is required, is occasional and temporary only, and is not likely to happen with more than one engine at

he same time; still we should have expected that this extra strain would have required extra strength, not only in the pinion and shaft, which receives the first shock, but also in a modified degree by all the rest of the gearing. The fact seems to be that the two large and heavy wheels E, E act as fly-wheels, absorbing and equalizing the extra strain which they occasionally receive, and transmitting to the rest of the gearing a moderately regular resistance.

It should be observed that while the calculated power of the two pinions F, F is $25\cdot37 \times 2 = 50\cdot74$-horse, that of the wheel E which drives them is only $25\cdot37$-horse by the same rules. The case of wheels driving two or more pinions is apparently anomalous and exceptional. It is investigated more fully in (71).

(148.) "*Water-wheel and Rag-engines, Foots Cray.*"—A very similar case to that we last considered, is shown by Fig. 54, where we have an overshot water-wheel, with 15 feet fall, driving four rag-engines, requiring $4 \times 6 = 24$ horse-power as before.

The iron-toothed wheels A, B were 10 feet $0\frac{1}{4}$ inch, and 5 feet 3 inches diameters, 3 inches pitch, $7\frac{1}{2}$ inches wide. The calculated power of A by the rules in (60) is $\sqrt{10 \times 5\cdot09} \times 3^2 \times 7\frac{1}{2} \times \cdot043 = 20\cdot7$-horse. The power they had to carry being 24-horse, was $24 \div 20\cdot7 = 1\cdot16$ or 16 per cent. in excess of the calculated power, but they gave no trouble in practice.

The mortise-wheels C, D were 9 feet $7\frac{9}{16}$ inches, and 5 feet $0\frac{3}{8}$ inch diameters, $2\frac{3}{4}$ inches pitch, 6 inches wide; hence the power of D $= \sqrt{5\cdot031 \times 18\cdot61} \times 2\frac{3}{4}^2 \times 6 \times \cdot05 = 22$-horse.

The mortise-wheel E, 13 feet $1\frac{1}{2}$ inch diameter, $2\frac{1}{4}$ pitch, 5 inches wide, drove two pinions F, F, $18\frac{5}{8}$ inches diameter, the calculated power of which is $\sqrt{1\cdot55 \times 158} \times 2\frac{1}{4}^2 \times 5 \times \cdot05 = 19\cdot8$-horse, and this of course would be the calculated power of the wheel E supposing that it drove only one pinion (71). But in our case we have two wheels E, E, and four pinions, the combined power $= 19\cdot8 \times 4 = 79\cdot2$-horse; but, as we have seen, the pinion D which drives the whole is 22-horse only. The explanation of this apparent anomaly will be the same as for the parallel case in (147).

(149.) "*2nd Water-wheel, &c., Foots Cray.*"—This case, shown by Fig. 55, is rather different to the last, as we have only two rag-engines instead of four; in other respects it was very similar. The water-wheel was precisely the same as the last, except that the width was reduced to about half, or in proportion to the power required, which now becomes 12-horse.

The iron-toothed wheels A, B were 9 feet $9\frac{1}{4}$ inches, and 5 feet $0\frac{5}{8}$ inch diameters, $2\frac{3}{4}$ inches pitch, 6 inches wide, $= 14\cdot3$-horse by the rules in (60). The mortise-wheels C, D were 9 feet $6\frac{5}{8}$ inches, and 5 feet $0\frac{1}{4}$ inch diameters, $2\frac{1}{2}$ inches pitch, $5\frac{1}{2}$ inches wide, and the power $= 17\cdot38$-horse. The wheels E, F were the same in all respects as before (148), and their power $= 19\cdot8$-horse, or $39\cdot6$-horse for the two pinions. In this case all the wheels were somewhat stronger than necessary, and they increased in strength progressively from the water-wheel to the rag-engines, being $14\cdot3$, $17\cdot38$, and $19\cdot8$-horse respectively. This seems to be right in principle where the work is of such a character as to cause violent temporary shocks, such as crushing and rolling mills, for although the gearing attached to the machine may take the brunt of the extra strain, and by its vis inertia may absorb much of it, still a portion must be transmitted to the intermediate gearing next following, and another and still smaller portion to the first-motion wheels.

(150.) "*Water-wheel and Rag-engines, Poyle.*"—In this case, shown by Fig. 53, we have an arrangement similar to the preceding, except that there were *three* pinions working in one driving wheel. A breast water-wheel 16 feet diameter, with a fall of 6 feet, drove three rag-engines equal to 18 nominal horse-power, the speed of the bucket being 5 feet per second.

The iron-toothed spur-wheels O, P were 12 feet 3 inches, and 3 feet $7\frac{3}{4}$ inches diameters, $2\frac{3}{4}$ pitch, $6\frac{1}{2}$ inches wide, and $18\cdot1$ horse-power by the rules in (60). The mortise-wheels R, S were 12 feet $0\frac{13}{16}$ inch, and $18\frac{5}{16}$ inches diameters, $2\frac{1}{2}$ inches pitch, $4\frac{3}{4}$ inches wide; hence the power of each rag-engine pinion was 23-horse, an excess of $23 - 6 = 17$-horse. The strength of the three pinions was $23 \times 3 = 69$-horse, but the strength of the pinion P which drove the whole was $18\cdot1$-horse only, or almost exactly the *mean* power required to do the

work. The explanation of this apparent anomaly has already been given in (145), &c., and need not be repeated here.

(151.) "*Steam-engine and Rag-engines, Dartford.*"—Fig. 57 shows the arrangement of a pair of double-cylinder or "Woolf" engines, collectively of 90 nominal horse-power, which drove fourteen rag-engines, allowing $6\frac{1}{2}$-horse for each.

The first-motion wheel A was an internal one 16 feet $8\frac{1}{2}$ inches diameter, 3 inches pitch, $8\frac{1}{2}$ inches wide, and at 22 revolutions its strength by the rules in (60) is $\sqrt{16 \cdot 7 \times 22} \times 3^2 \times 8\frac{1}{2} \times \cdot 05 = 73 \cdot 3$ horse-power, whereas the power of the engines was 90-horse. But, as shown in (71), the case of a wheel driving two or more pinions is exceptional, and in our case, although its power with one pinion would be 73·3-horse, with two pinions it would be $73 \cdot 3 \times 2 = 146 \cdot 6$-horse. On strictly correct principles, the strength of the teeth should have been calculated for 45-horse, and that of the arms (73) for 90-horse; but in that case obviously, the wear-and-tear with two pinions would be double that with one, and for that reason, possibly, an intermediate strength of 73-horse was adopted; by the rule in (72) it comes out 64-horse. The two lines of shafting were driven by the mortise-pinions B, B, 4 feet $1\frac{5}{8}$ inch diameter, &c.

As shown in (70), the wheels for rag-engines require to be of extra strength, the mortise-wheels C, C, C were placed in pairs, each driving one rag-engine, C was 4 feet 7 inches diameter, $2\frac{1}{2}$ inches pitch, 5 inches wide, and its power with 88 revolutions by the rule in (60) is $\sqrt{4 \cdot 58 \times 88} \times 2\frac{1}{2}^2 \times 5 \times \cdot 05 = 31$-horse, and as the mean power was say 7-horse, we have an excess of $31 - 7 = 24$-horse.

(152.) The proper sizes for the shafting may now be calculated on the principles governing exceptional cases explained in (89) and elsewhere. Say that we admit for safety that with seven rag-engines in one line, two at the remote end are taking the maximum power due to their pinions, or $31 \times 2 = 62$-horse, and the rest, the normal amount of 7 horse-power; then the shaft G must be calculated for 62 horse-power by the rule for *large* diameters in (75); taking the value of **M** for wrought iron at 160, we obtain for G $(62 \times 160 \div 88) \sqrt[3]{} = 4\frac{7}{8}$ inches dia-

meter; then F having to drive two extra rag-engines, the power becomes $62 + 14 = 76$-horse, and the diameter $(76 \times 160 \div 88) \sqrt[3]{} = 5\frac{3}{16}$ inches; for E we have $76 + 14 = 90$-horse, and $(90 \times 160 \div 88) \sqrt[3]{} = 5\frac{1}{2}$ inches diameter; and finally for D, $90 + 7 = 97$-horse, and $(97 \times 160 \div 88) \sqrt[3]{} = 5\frac{5}{8}$ inches diameter. The actual sizes were a little less than we have calculated, but there was a fly-wheel H at the remote end of each shaft which would tend to equalize the strain materially. The rag-engine pinion J was 2 feet $6\frac{1}{4}$ inches diameter, and made 160 revolutions per minute.

(153.) "*Steam Corn-mill, Taganrog.*"—This mill was constructed by Fairbairn, and consisted of thirty-six pairs of stones in one line, but as it was driven by engines placed in the centre, and driving right and left, we may take it that there were virtually eighteen pairs driven by one line of shafting. The engines were of the reputed nominal power of 200 horses, but gave 600 gross indicated, or, by our standard (4), 300 nominal horse-power; each shaft, therefore, carried 150-horse, and by the rule in (75) the diameter for cast iron with 254 for the value of **M** and 87·7 revolutions per minute, comes out $(150 \times 254 \div 87 \cdot 7) \sqrt[3]{} = 7\frac{5}{8}$ inches, or $\frac{5}{8}$ inch less than the actual size, which was $8\frac{1}{4}$ inches next the engine, reduced down successively to $7\frac{1}{4}$, $6\frac{1}{2}$, and finally to 6 inches at the remote end.

The work done by the eighteen pairs of stones was 100 bushels of wheat per hour, and we may calculate the proper diameter for the shaft by the principles indicated in (88), &c. To grind 100 bushels of wheat requires 100-horse indicated, or, by our standard (4), $100 \times 2 \div 3 = 67$-horse nominal; to dress 100 bushels requires 25-horse indicated, or, 17-horse nominal; the bolter and sack-tackles would take about 10-horse. Allowing, as in (70) that the *excess* of power is $13 - 3 = 10$-horse per pair of stones, and that six out of the eighteen pairs were taking that extra power, and the rest the mean normal power, we have $67 + 17 + 10 + (6 \times 10) = 154$-horse nominal, or nearly the same as before. We have seen in (142) that it is a critical arrangement to drive a number of millstones by one long shaft, and that sufficient stiffness cannot be obtained except by adopting sizes larger than the rules give, even after making all the allowances for known exceptional circumstances.

(154.) "*General Remarks.*"—It will be observed from the numerous examples we have given in this chapter and elsewhere throughout the work, that care and judgment are necessary in applying the rules so as to include *all* the conditions of the case.

For instance, if we required the diameter for a shaft with a given power and speed, we must, 1st. Ascertain if possible the real nominal horse-power by our standard (4) from the *net* indicated power (6), or from the *gross* indicated power (7), or from the size of cylinder, &c., &c. (8), (10). 2nd. Find if that power is uniform as from a water-wheel, or variable as due to the character of the prime mover (110), or to the action of the crank (76), or to the expansive action of the steam (86). 3rd. Observe if the work to be done is uniform as in driving a set of 3-throw pumps, or variable as in driving a circular saw, &c. (87). 4th. Consider torsional *strength* (75). 5th. Torsional *stiffness* (79). 6th. Transverse *strength*, where a wheel or crank overhangs the bearing (104). 7th. Transverse *stiffness*, if the shaft is heavily loaded and the bearings necessarily distant from the load (104). 8th. Abrasion, if the bearings are heavily loaded (97); and 9th. Heating, if the shaft is loaded and its speed considerable (99).

It should also be observed that when the work is very variable, the power of the engine and the strength of the gearing must usually be equal to the *maximum* strain, except indeed in cases where that strain is momentary only, and the momentum of the fly-wheel may suffice to overcome it; but even in that case, the gearing must be adapted to the maximum strain. Say that we have a clay-mill worked effectively by an ordinary horse walking in a circle for 10 hours per day, and we wish to substitute a water-wheel or steam-engine to do the same work. Now, by Table 4, the power of the horse is ·475, say $\frac{1}{2}$ horse net indicated, and it would appear that a water-wheel or steam-engine of that power would suffice. But, from the nature of the work, there are frequently times (when, for instance, a large lump of clay is thrown in) when the horse is obliged to exert his full strength for a short time, or by (14) about $\frac{1}{2} \times 6 = 3$-horse net indicated power; hence the water-wheel or steam-engine and the gearing must be equal to that power, or failure would result.

Cases where the work is more or less variable are very numerous (87), but where the variation is not great, special provision by extra strength of gearing is not necessary, because, as shown in (147), &c., our rules allow a certain margin of extra strength for contingencies. Thus in (144) we had wheels $12 \cdot 74 \div 16 = \cdot 80$, or 80 per cent. only of the strength required by the rule, or 20 per cent. too weak; they were therefore strained $16 \div 12 \cdot 74 = 1 \cdot 26$, or 26 per cent. beyond their calculated strength, yet they stood well. In (138) we had wheels strained 23 per cent. in excess of their calculated strength, which were satisfactory in practice; with even 50 per cent. in excess, the wheels in (147) did not immediately break down, indeed they did their work for some years, although not satisfactorily; but evidently, with that excess, failure is eventually certain, and is only a question of time.

As to shafting,—the 30-horse shaft in Table 15 which failed with 3 inches diameter to do work for which $3\frac{3}{4}$ inches diameter was required by the rule, was strained $3\frac{3}{4}^4 \div 3^4 = 2 \cdot 44$, or 144 per cent. above its normal strength, and failed, as might have been expected. When altered to $3\frac{1}{2}$ inches diameter, it was still $3\frac{3}{4}^4 \div 3\frac{1}{2}^4 = 1 \cdot 32$, or 32 per cent. overstrained, and although it did its work, it lacked the necessary stiffness, and was not perfectly satisfactory. The 'Warrior' shaft (85), whose actual strength was 53 per cent. only of that required by our rules, and was strained $21^3 \div 17^3 = 1 \cdot 88$, or 88 per cent. above its normal strength, seems to bear even that excessive strain.

Generally, we may admit, that for perfect safety and permanence the strains given by our rules for Wheels and Shafts should not be exceeded, but that on emergency, 25 per cent. of extra strain may be allowed *constantly* without much risk of failure: for *occasional* and temporary variations of strain, perhaps 40 per cent. in excess may be permitted, but beyond those two limits, the case must be regarded as exceptional (69), (85), and extra strength of gearing must be provided to bear the extra strain.

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·18	3·29	3·42	3·55	3·68	3·81	3·95
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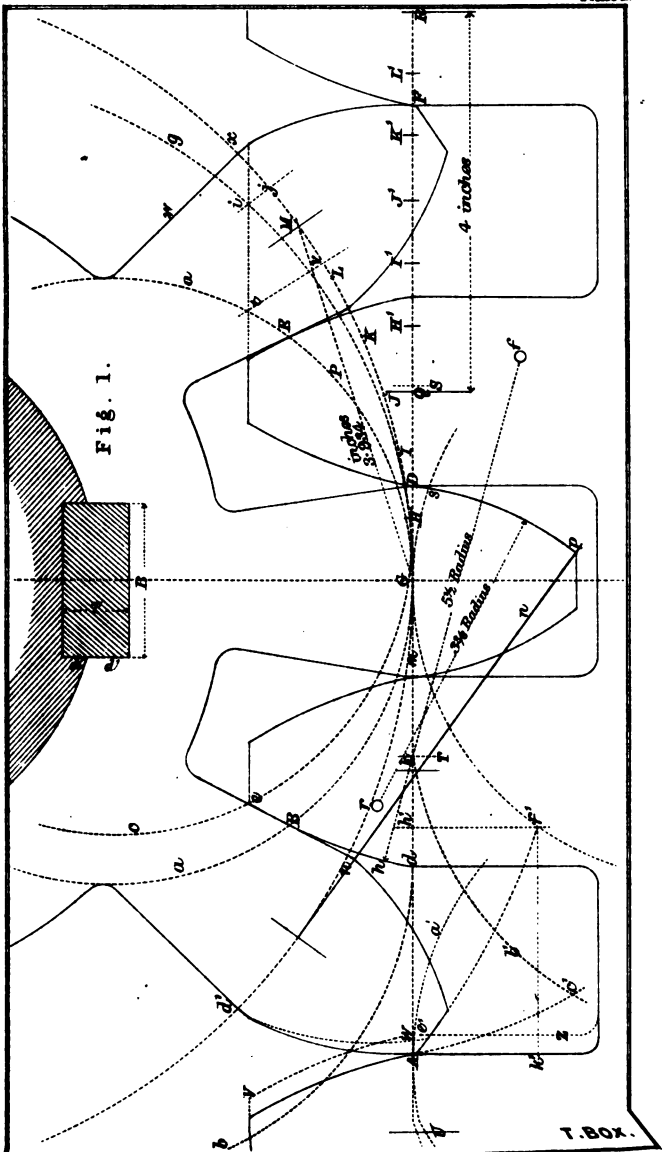

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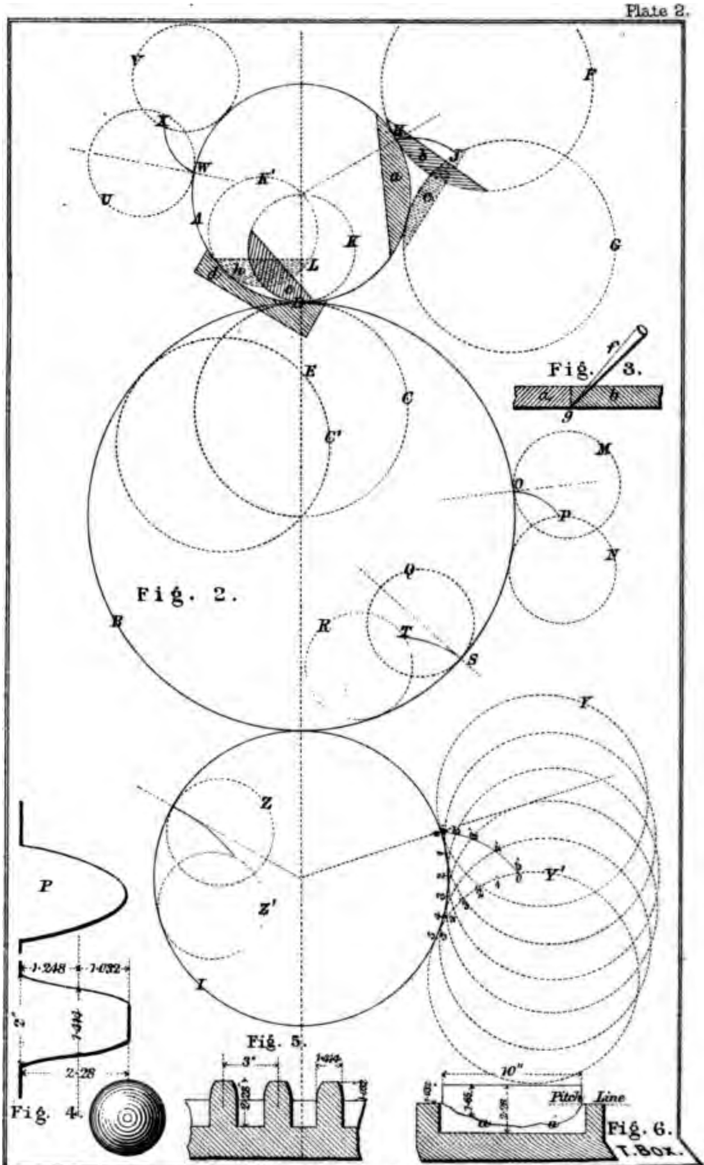

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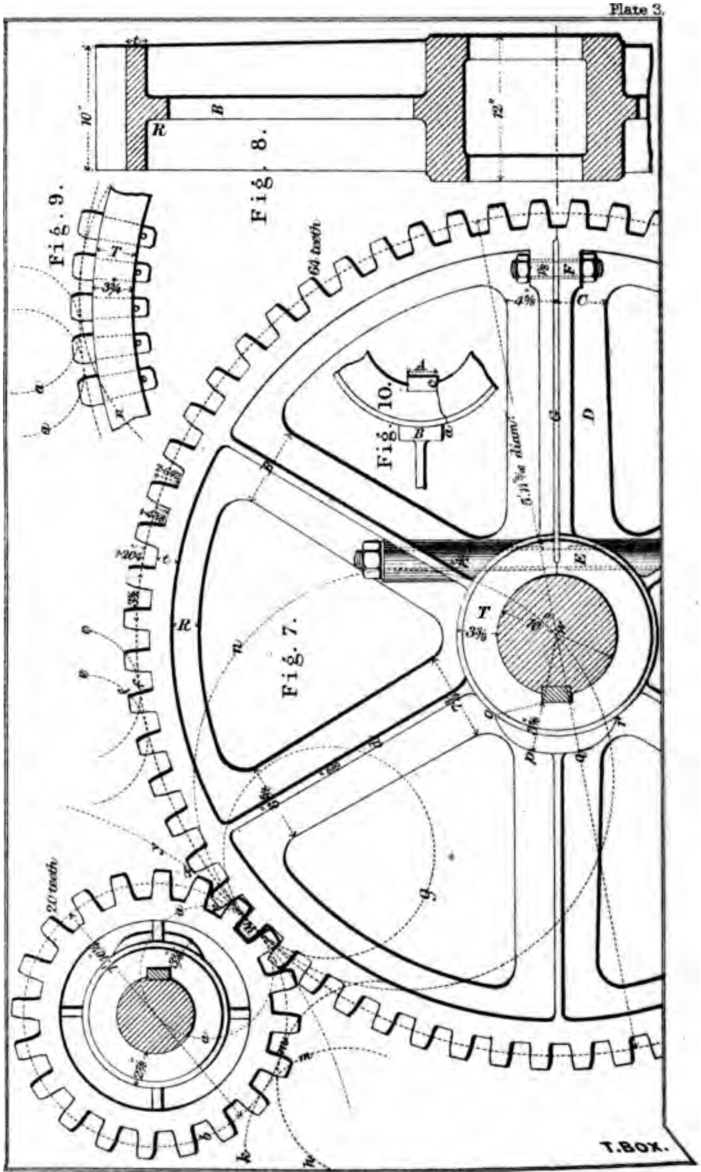

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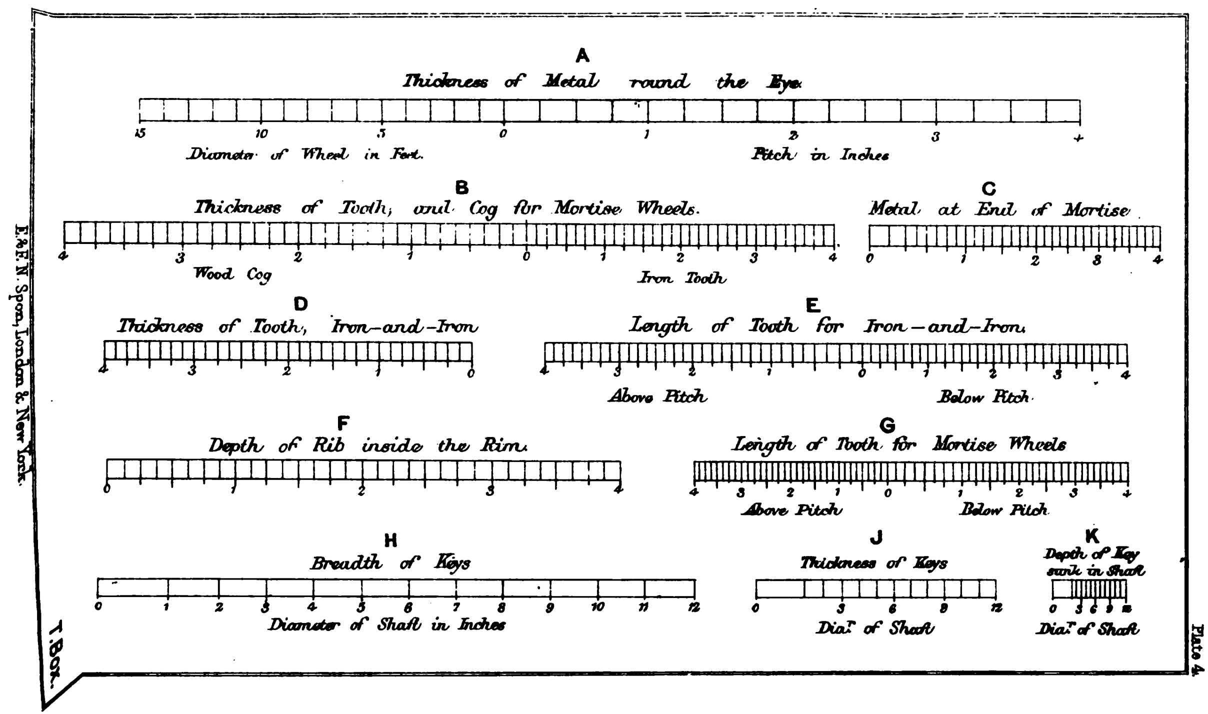
Plate 4.
A
Thickness of Metal round the Eye
15 10 5 0 1 2 3 4
Diameter of Wheel in Feet.
Pitch in Inches
B
Thickness of Tooth, and Cog for Mortise Wheels.
4 3 2 1 0 1 2 3 4
Wood Cog
Iron Tooth
C
Metal at End of Mortise
0 1 2 3 4
D
Thickness of Tooth, Iron-and-Iron
4 3 2 1 0
E
Length of Tooth for Iron-and-Iron
4 3 2 1 0 1 2 3 4
Above Pitch
Below Pitch
F
Depth of Rib inside the Rim
0 1 2 3 4
G
Length of Tooth for Mortise Wheels
4 3 2 1 0 1 2 3 4
Above Pitch
Below Pitch
H
Breadth of Keys
0 1 2 3 4 5 6 7 8 9 10 11 12
Diameter of Shaft in Inches
J
Thickness of Keys
0 3 6 9 12
Diar. of Shaft
K
Depth of Key sunk in Shaft
0 3 6 9 12
Diar. of Shaft
T. Box.
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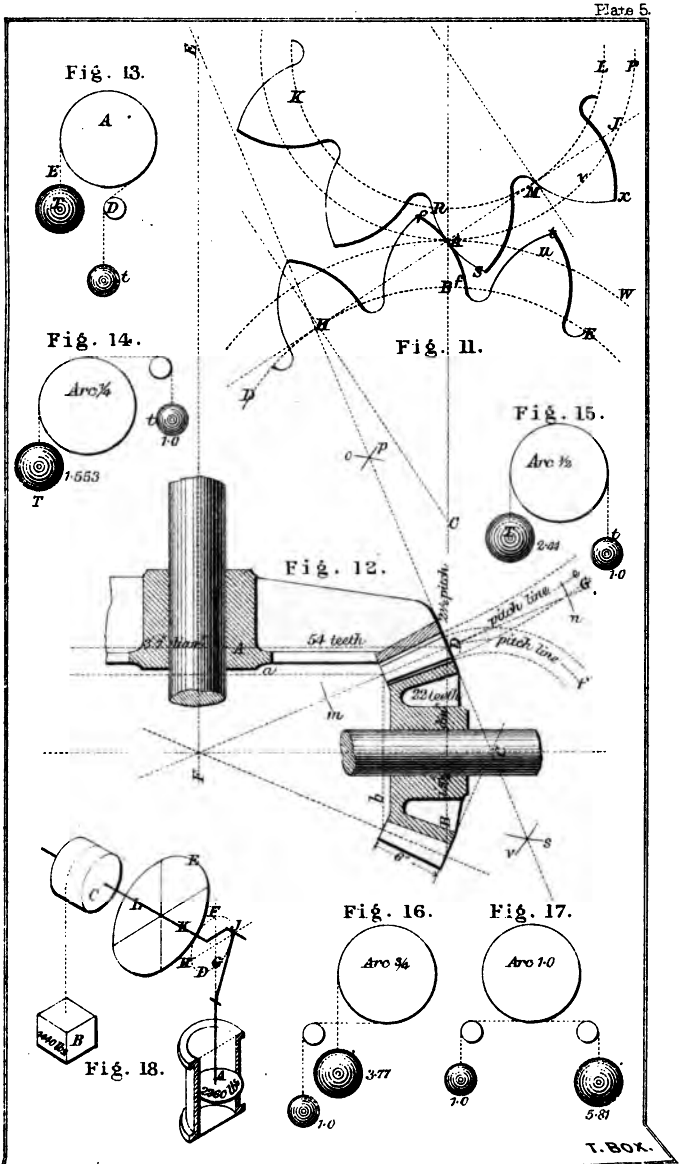

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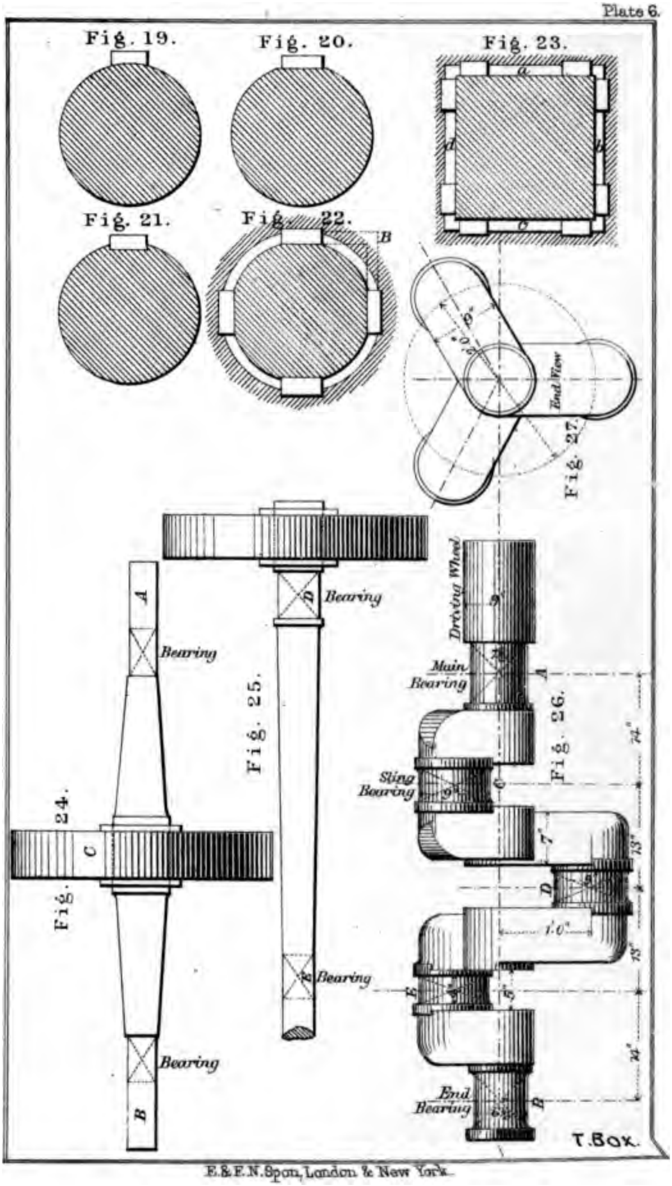

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Fig. 28.

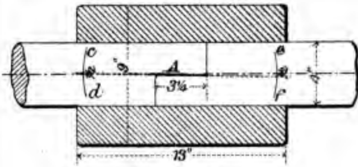


Fig. 29.

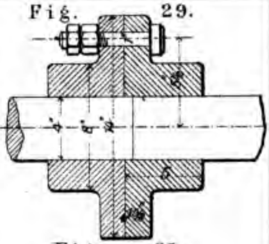


Fig. 30.

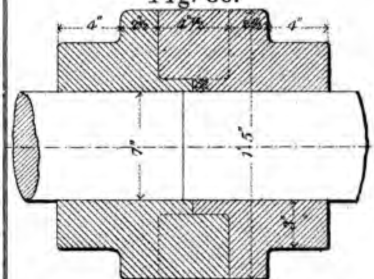


Fig. 31.

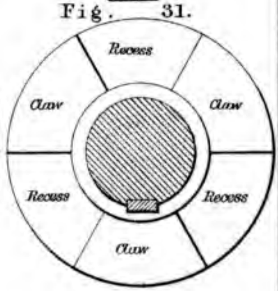
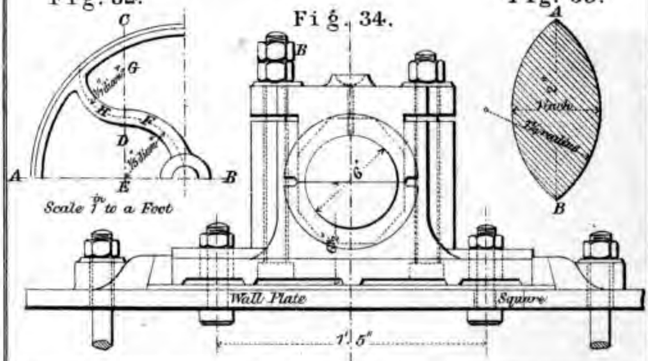


Fig. 32.



Scale 1 in to a Foot

Fig. 34.

Fig. 33.

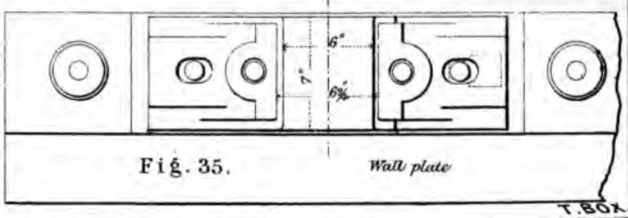


Fig. 35.

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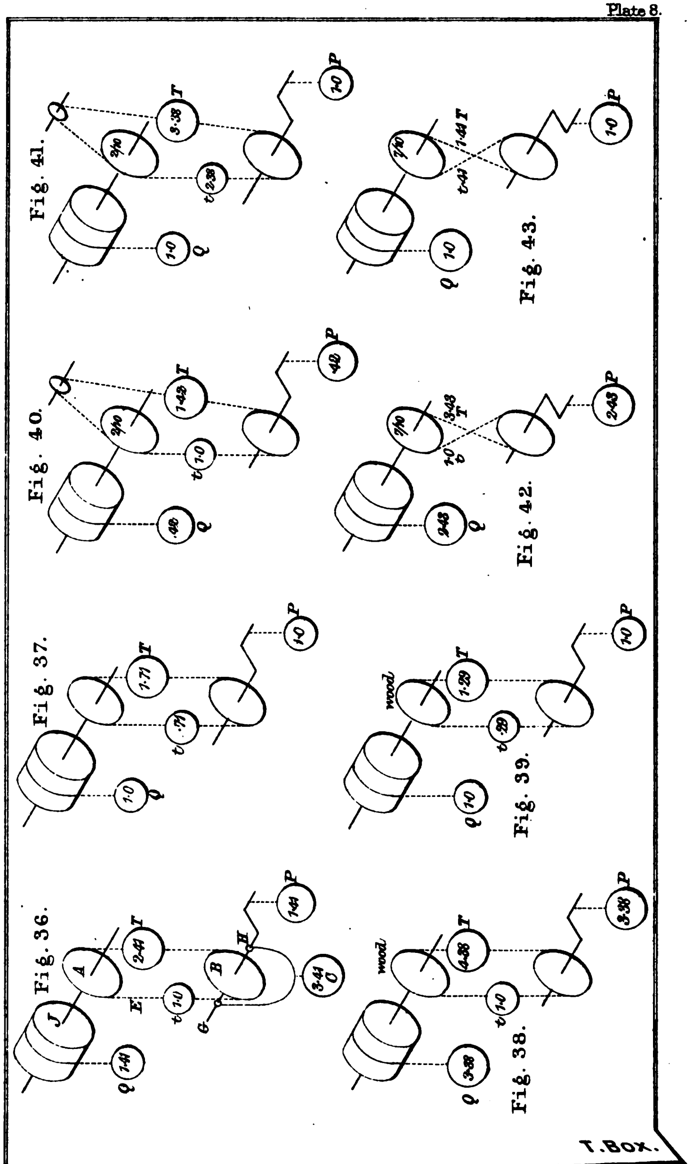
Plate 8.



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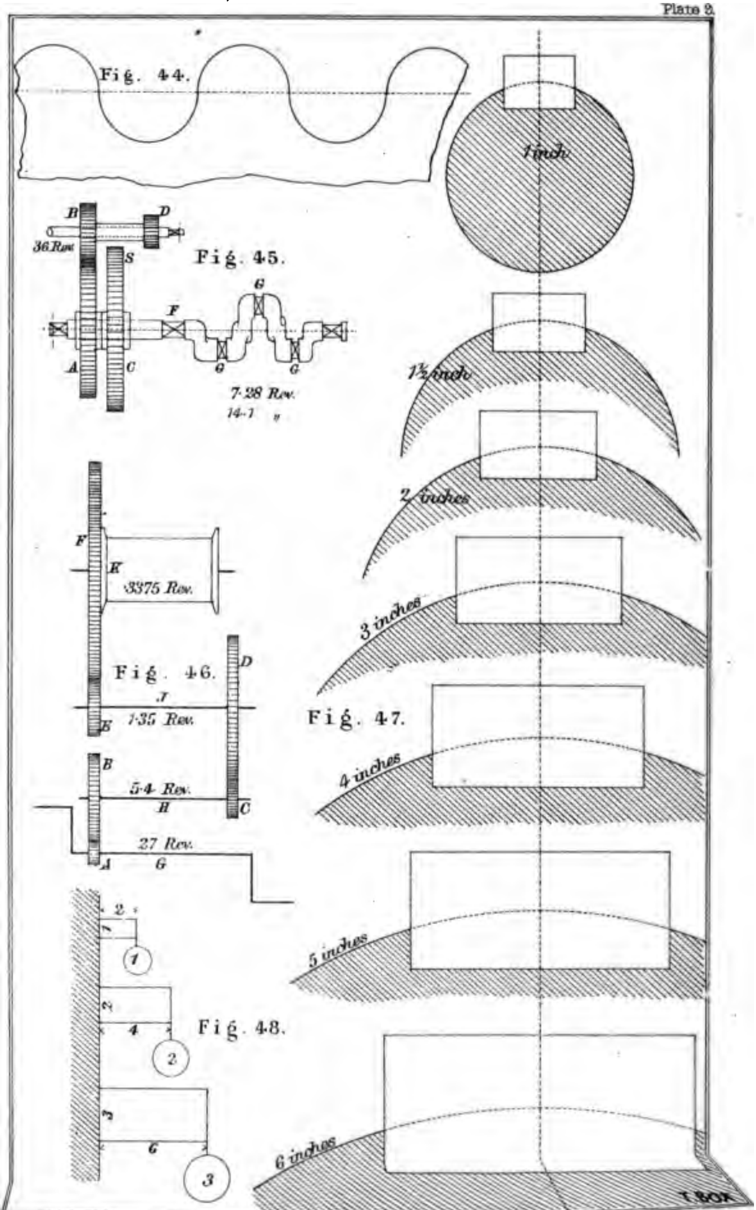

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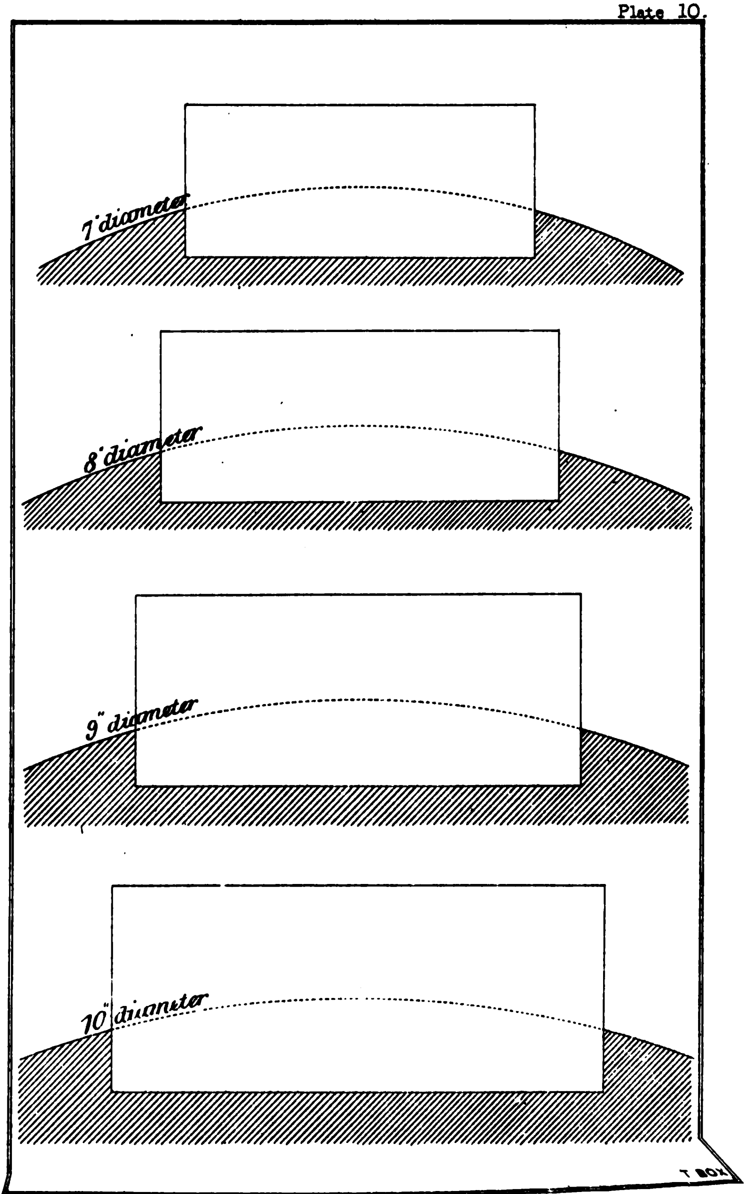

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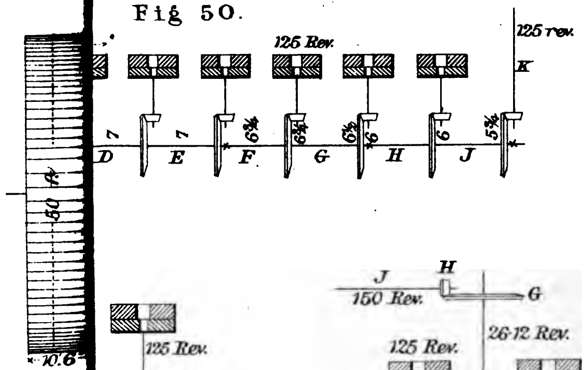
Fig. 50.



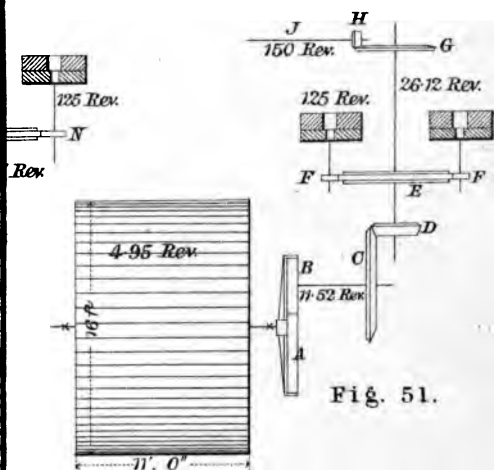


Fig. 51.

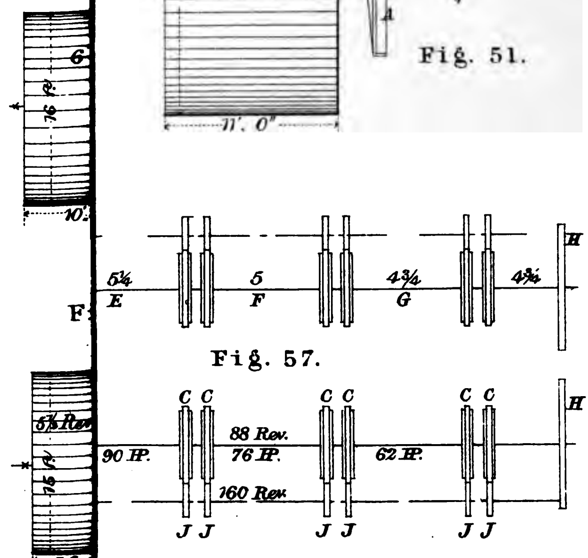


Fig. 57.

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